PHAS1511 – problem sheet 2 answers

1. What do we mean when we talk about a *black body*? What does the spectrum of a black body look like? Do the spectra of stars resemble those of black bodies? Do the spectra of nebulae resemble those of black bodies? (4)

A black body is one which absorbs all the radiation that falls on it. The radiation it emits is defined only by its temperature – that is, any black body behaves in the same way, regardless of what it's made of.

The spectrum of a black body is a smooth curve with a peak whose position depends on the temperature of the black body.

Dense, opaque bodies have spectra that are similar to black bodies. Stars are dense and opaque and so their spectra resemble those of black bodies. Nebulae have extremely low densities and are not opaque, so their spectra do not resemble those of black bodies.

2. When Ernest Rutherford fired alpha particles at very thin gold foil, what happened? What did this tell us about gold atoms? A cloud of hot hydrogen gas emits radiation at certain particular fixed wavelengths. What does this tell us about the nature of hydrogen atoms? (6)

Surprisingly, most of the alpha particles passed straight through with little or no deflection. A few, though, had very large deflections. This means that gold atoms must consist mostly of empty space, with all of their mass concentrated in a very small area. This is found to be true of all atoms – they consist of a very small dense *nucleus,* surrounded by electrons.

Hydrogen emits radiation at certain wavelengths because electrons cannot orbit the nucleus at any distance but only in certain fixed orbits. When an electron moves between fixed orbits, the energy change has a fixed value, and so radiation is emitted at certain fixed wavelengths.

3. What observations of a star do we need to measure its motion along the line of sight? What observations do we need to measure its motion in the plane of the sky? (3)

We need spectroscopic observations to measure the motion of stars along the line of sight. By observing spectral lines whose wavelength in the laboratory we know, we can measure the Doppler shift and therefore the velocity of the star.

To measure motions in the plane of the sky, we need two images separated by a long enough time to detect the very slow apparent motion of the star across the sky. The time may be a year or two for the closest, fastest-moving stars (such as Barnard's Star), but will be longer for more distant or slow-moving objects. To convert apparent motions into actual velocities, we also need to know the distance of the star.

4. The William Herschel Telescope in the Canary Islands has a mirror 4.2m in diameter. What is the smallest angular separation that the WHT could measure, theoretically? Why might it not reach this limit in practice? (2)

The diffraction limit of a telescope, in arcseconds, is given by 2.5×10^{-4} λ/D, where λ is the wavelength you're observing in nanometres, and D is the diameter of the telescope in metres. So, if we were looking at wavelengths of 550nm with the WHT, we could resolve objects separated by 0.03 arcseconds.

In practice, the WHT mirror might not be perfectly figured to the right shape, and objects it observes are smeared out by the movement of the atmosphere.

5. The WHT is a reflecting telescope. Give three reasons why it would be impractical to build a refracting telescope this large. (4)

1. A 4.2m lens would be extremely heavy – too heavy for a telescope tube to support without flexing.
2. A 4.2m lens with glass of sufficient quality for astronomical use would be extremely difficult and thus expensive to manufacture
3. Refracting telescopes suffer from chromatic aberration; correcting it requires a second or even third layer of glass in your lens – impractical for very large lenses.

6. The WHT is designed to observe visible light, with a typical wavelength of 550nm. If you wanted to observe radio waves with a wavelength of 50cm, at the same resolution as the WHT has in the optical, how large a radio telescope would you need? (2)

50cm is 909,000 times larger than 550nm, and therefore you would need a telescope 909,000 times larger than the WHT – 3.8km across - to observe 50cm radio waves.

7. When stars are plotted on a graph of luminosity against temperature, most stars lie on the *main sequence*. What determines where on the main sequence a star will lie? (1)

Only its mass – heavier stars are brighter and hotter and lie towards the top left of the graph, while lighter stars are cooler and fainter, and lie towards the bottom right.

8. The Crab Nebula formed in a supernova explosion. Photographs show that in 1973 it had a size of 276 arcseconds, while in 2008 it has a size of 287 arcseconds. Roughly when did the explosion occur? (2)

In 35 years is has expanded by 11 arcseconds, giving a rate of 0.31 arcseconds/year. To reach a size of 287 arcseconds at this speed would take 913 years, so by this reckoning it exploded in about 1095AD. (The actual date of the explosion was 1054AD – the star became bright enough to be visible in broad daylight, and was recorded by Chinese and Arab astronomers)

9 a) The star Betelgeuse in Orion has a parallax of 5.07 milli-arcseconds. What is its distance in parsecs? (1)

Distance in parsecs = 1 / parallax in arcseconds, therefore its distance is 197 parsecs

b) If we carefully measure the amount of energy we receive at Earth from Betelgeuse, we find that it is $1.3 \times 10^{-7} W/m^2$. What amount of energy is being emitted by Betelgeuse? The solar luminosity is 3.89×10^{26}W – how much more luminous than the Sun is Betelgeuse? (4)

The distance of Betelgeuse is 197 parsecs, which is equal to 6.08×10^{18} metres. The energy we receive from Betelgeuse here is equal to the energy it emits, spread out over the surface of a sphere with a radius equal to the distance between here and there. The surface area is $4\pi R^2 = 4.65 \times 10^{38} m^2$, and so multiplying this by the energy we receive gives us the energy emitted: 6.03×10^{31} W.

If the solar luminosity is 3.89×10^{26}W, then Betelgeuse is 155,000 times as luminous as the Sun.

c) If we look at the spectrum of Betelgeuse and find that the wavelength of the peak of the emission is 850nm, what is the temperature of Betelgeuse? (1)

Wien's law relates the peak wavelength of a black body spectrum to its temperature: $T=2,900,000/\lambda$. Assuming Betelgeuse is a black body, then T = 2,900,000/850 = 3,412K

d) What is the radius of Betelgeuse in metres? The Sun's radius is 700,000 kilometres – how much larger than the Sun is Betelgeuse? (4)

To answer this, recall that the luminosity of a star is related to its temperature and radius by $L = 4\pi R^2 \sigma T^4$, where σ is Stefan's Constant. We now have Betelgeuse's luminosity and temperature, so its radius is $\sqrt{(L/4\sigma\pi T^4)} = 7.9 \times 10^{11}$ m. This is 1,130 times larger than the Sun.