The Michelson Interferometer

Experiment Aims & Objectives:
To set up and use a Michelson Interferometer to measure

- the wavelength separation of the sodium $D$ lines
- the thickness of a sample of Mylar

Your write-up should explicitly address questions in the script flagged as ‘Q1–3’, and summarize your results from working through Section 3. It should not reproduce large chunks of this script, but should include fully annotated tables of measurements, and sufficient details of calculations for your working to be readily followed.

1 Background

The Michelson interferometer, developed by Albert Michelson (right) in 1881, is one of the best known of optical instruments used by physicists and astronomers. Over the years it has been adapted to work in many wavelength regimes and in many specialised forms for purposes such as length measurement, high-resolution spectroscopy, optical-component testing, etc. It played an important role in the development of special relativity (the Michelson–Morley experiment), and in the first measurements of the angular sizes of stars. Most recently, it is the fundamental tool at the heart of the LIGO experiment, which provided the first direct detection of gravitational radiation.

The instrument used in this experiment is a commercially available laboratory version of the original Michelson layout, shown diagrammatically in Fig. 1.

2 Physical principles

A beam of radiation from the source $S$ (Fig. 1) is divided into reflected and transmitted beams of approximately equal intensity by the thin semi-reflecting beam-splitter, $A$. (In practice, a thick beam-splitter is used, which introduces a complexity, discussed below.) The two beams are then reflected back by the (front-silvered) mirrors $M_1$ and $M_2$ and recombine again at the surface $A$. The resulting interference pattern caused by the difference in the path lengths between the two arms of the interferometer is viewed in the direction shown.
2.1 Circular Fringes with Monochromatic Light.

Let $M'_2$ be the image of $M_2$ formed by reflection in the beam splitter. The path difference travelled by the beams in the two arms of the interferometer is then equivalent to the distance $2M_1M'_2$. If $M'_2$ is parallel to $M_1$ then for a beam inclined at an angle $\theta$ to the normal to $M_1$ this path difference will be

$$\Delta L = 2d \cos \theta$$  \hspace{1cm} (1)

where $d$ is the distance $M_1M'_2$.

**Q1 – with the aid of a sketch, explain why this is so.**

The resultant phase shift (expressed in radians) between the recombined beams at wavelength $\lambda$ is

$$\delta = \frac{2\pi}{\lambda} 2d \cos \theta$$  \hspace{1cm} (2)

For a diffuse source, giving rise to a wide range in $\theta$, there will be maxima and minima in the intensity of the interference pattern corresponding to the conditions

maxima: $\delta = 2m\pi$  \hspace{1cm} (3)

minima: $\delta = (2m + 1)\pi$

where $m$ is an integer known as the ‘order of interference’.

The interference pattern therefore consists of a number of bright and dark fringes, which are circular about the normal to $M_1$: Each fringe subtends an angle $\theta_m$ such that

$$\cos \theta_m = \frac{m\lambda}{2d}$$  \hspace{1cm} (4)

(obtained by rearranging eqtns (2) and (3)).
If each beam is of equal amplitude $a$, then the recombined beam will have an intensity in direction $\theta$ given by

$$I(\theta) = 4a^2 \cos^2\left(\frac{\delta}{2}\right)$$

The $\cos^2(\delta/2)$ variation in intensity gives rise to fairly diffuse fringes; this is typical of interference patterns formed when only two beams are involved.

If $M_1$ is initially set so that $d$ is large and is then moved so that $d$ becomes smaller, then for any given $m$ the fringe must move inwards (to increase the value of $\cos \theta$), towards the centre of the pattern where it will eventually disappear. One fringe will disappear and each fringe will move exactly 1 interval for each decrease of $\lambda/2$ in the spacing $M_1 M_2$. As $d$ gets smaller the separation of the fringes will get wider and eventually, when $d = 0$, the central fringe will expand to cover the entire field:

Fringes formed for (left to right) $d$ large; small; zero (‘zpd’); small; large.

It might be supposed that the zero-path-difference (zpd) fringe should be bright, due to constructive interference. In fact, due to the internal reflection in the beam splitter and the external reflection at the beam splitter, the conditions in the two light paths differ, and an additional phase change of $\pi$ is introduced, so that in practice the zpd intensity is a minimum, i.e., a dark fringe. As $d$ begins to increase again, after passing through the zpd position, the fringe system begins to shrink as shown in the last two panels above.

**Q2 – Where exactly does this extra phase change of $\pi$ occur?**

### 2.2 Doublet Source

For light sources having two monochromatic wavelength components $\lambda_1$ and $\lambda_2$ differing by a small amount $\Delta \lambda$, such as the orange-yellow sodium $D$ lines, there will be two fringe systems superimposed which are very similar but not identical. For some mirror positions, where $2d$ is an integer number of wavelengths for both components, the fringes will coincide over a good range of $\theta$ and will look sharp (that is, have maximum visibility). In other positions of $M_1$ the maxima of one fringe system will coincide with the minima of the other and the two patterns will add to give roughly uniform intensity (minimum fringe visibility). As $M_1$ is moved the fringe visibility will go through alternate cycles of maximum and minimum visibility, and the wavelength difference $\Delta \lambda$ can be determined from the separation of these cycles.
From the foregoing reasoning, the value of $d$ giving maximum visibility of the two fringe systems is

$$2d = m_1\lambda_1 = m_2\lambda_2$$

(6)

where $m_1$, $m_2$ are integers; that is,

$$m_1 - m_2 = 2d \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right).$$

(7)

If $d$ is changed to $d + \Delta d$, corresponding to the next cycle of maximum visibility, then $(m_1 - m_2)$ must have changed by 1, so that

$$m_1 - m_2 + 1 = 2(d + \Delta d) \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right).$$

(8)

Subtracting eqtn. (7) from (8) and rearranging gives

$$\Delta \lambda = \frac{\lambda_1\lambda_2}{2\Delta d} \approx \frac{\lambda_m^2}{2\Delta d}$$

(9)

where $\lambda_m$ is the mean wavelength of the doublet (since $\lambda_1$ and $\lambda_2$ are almost equal).

[The same considerations apply, and the same formula results, if $\Delta d$ is determined as the distance between positions of minimum visibility.]

### 2.3 White-light fringes

If the mirrors $M_1$ and $M'_2$ are inclined at a very small angle and $d$ is small, then the fringes formed will be characteristic of those formed by a thin optical wedge. With an extended monochromatic source the fringes will be nearly straight lines parallel to the line of intersection of $M_1$, and will be localised in the plane of the wedge. For small $d$ these fringes represent increments of $\lambda/2$ in the wedge thickness.

[Achieving this condition needs care in practice, as it is very sensitive to adjustment of $M_2$ when the fringes are wide.]

![Figure 2: White-light fringe visibility: (a), small $d$; (b), larger $d$.](image)

For larger $d$ there is an additional $\cos \theta$ variation in the path length and the fringes become curved, as shown in Fig. 2.

The zeroth-order fringe formed at the line where $M_1$ and $M'_2$ intersect should be bright but, owing to phase shift of $\sim 180^\circ$ introduced by the beam splitter, this fringe is dark in practice.
Figure 3: The experimental arrangement when aligned.

If white light is used to observe the region around zero path difference ($d = 0$) then the intersection of $M_1$ with $M'_2$ is indicated by a dark fringe. On either side of this there will be a few coloured fringes, and in the outer field (where the different coloured fringe systems overlap completely) the fringes disappear into a white continuum. The importance of the central dark fringe lies in the fact that it indicates the exact position where $M'_2$ intersects $M_1$, and therefore provides a very sensitive indicator of the position of $M_1$.

3 Experiment

[Remember, no determination of a physical quantity is complete without an assessment of its uncertainty. Think about how you will assess the uncertainty before you take any measurements and hence make sure that you have enough information to make a sensible assessment.]

The experimental arrangement is shown in Fig 3. The orientation of $M_2$ can be adjusted for alignment with $M_1$ and the path length in the reflective arm can be altered by moving $M_1$ using a micrometer screw and lever arm. Note that the interferometer includes an extra component which is not shown in Fig. 1: the compensating plate. This extra plate is necessary because the beam-splitter unit is a thick glass plate whose back surface is lightly aluminised. (It is this thin aluminium layer which does the actual beam splitting; the glass is just a suitable medium to support it.) The thickness of the glass adds an extra, wavelength-dependent optical path into the reflective arm of the interferometer. To equalise the path length in both arms a compensating plate of exactly similar material and thickness is placed in the transmission arm. This is important when white-light fringes are required.

Q3 – How many times do the two main interfering beams (shown as strong dark lines in Fig. 3) traverse a path in glass? Is it the same in each of the arms?
3.1 Adjustment for circular fringes.

Use a mercury-discharge lamp as source, with a green filter to isolate the 546.07nm mercury line. Adjust the lamp and lens to illuminate the diffuser brightly and evenly. Replace the diffuser with the pinhole provided and look into the interferometer in the direction indicated. Several images of the illuminated pinhole will be seen. Some of these are due to reflections from the un-aluminised face of the beam splitter and $M_1$ and $M_2$ (light paths shown dotted in Fig 3), but the two brightest involve reflections from $M_1$ and $M_2$ and the real beam-splitter surface. $M_2$ and $M_1$ are not aligned at this point.

![Image](a) ![Image](b) ![Image](c)

Figure 4: White-light fringe visibility

To make $M_1$ and $M'_2$ parallel, $M_2$ must be tilted slightly using the adjustment screws on the back of its mount until the two brightest images of the pinhole overlap completely. If the pinhole is now replaced with the diffusing screen, fine fringes at a random orientation in the field of view should now be visible by relaxing the eye (i.e., looking at infinity) and further careful adjustment of $M_2$ should enable them to be centred in the field of view and made more distinct (Fig. 4a, b).

For the final adjustment, $M_1$ should be moved by use of the micrometer drive to make $d$ small and the fringe diameter large (Fig. 4c).

[Slight adjustment of $M_2$ may be needed to keeping the fringe system central in the field of view.]

3.2 Calibration of the mirror movement.

The length of the lever arm coupling the micrometer screw with $M_1$ is unknown and the relationship between the micrometer movement $\Delta L$ and the corresponding mirror shift $\Delta d$ must be calibrated. In general, a linear relationship is assumed:

$$\Delta d = f \Delta L$$

where $f$ is a correction factor.

You should determine $f$ by counting the number of fringes passing a point in the field of view as the micrometer drum is slowly turned. A click counter is provided to help you keep count. Each fringe represents a mirror shift of $\lambda/2$, and it will be necessary to count at least 200(!) fringes to obtain an accurate value of $f$. Repeat the calibration for at least 3 well-spaced positions along the micrometer screw, taking care to avoid errors due to backlash. Unless the results differ significantly, the mean value of $f$ can be taken to apply to all micrometer readings. You should, of course, estimate an error on the value of $f$. 
When counting fringes it may help to have a fine pointer (a wire or pin) mounted so that it is visible in the field along with the fringes or use can be made of the central fringe since it is so distinctive in appearance. Either method will assist as they will serve as a reference point in space for the eye. (Confirmation that you have performed the calibration correctly in measuring a value for this factor $f$ can be made by calculating the ratio of the distance from the pivot to the point where the rod from $M_1$ touches the lever and the distance from the pivot to the point of application of the micrometer drive onto the lever. This will provide a check that your value is about right.)

3.3 Separation of the sodium doublet lines.

Replace the mercury lamp with the sodium discharge lamp, and observe how the visibility of the orange-coloured fringes varies as $M_1$ is moved. You will note that the visibility varies as the two fringe systems go in and out of phase. Measure the position of $M_1$ in as many positions of maximum (or minimum) visibility as you can manage. Deduce the wavelength separation of the two lines from the mean interval between successive cycles. Also evaluate the standard deviation and the error on this mean. You may assume the mean wavelength of the doublet to be 589.3 nm.

3.4 Adjustment for white-light fringes.

Using the sodium lamp as the source of illumination, adjust the position of $M_1$ to give large diameter fringes; i.e., make $d$ small to make the centre fringe almost fill the field of view. Tilt $M_2$ slightly so that ∼5–10 curved fringes appear across the field of view. As $M_1$ is moved, the fringe curvature will change, becoming straight and then reversing as $M_1$ is moved through the position of zero path difference. Set $M_1$ as close as you can judge to this position of straight fringes, and replace the monochromatic light source with the white-light incandescent lamp provided. If $M_1$ is now slowly moved away from its set position the white-light fringes should appear as a distinctive set of coloured fringes with a dark central fringe indicating the point of zero path difference, with a bright fringe either side of it. These fringes are localised at $M_1$ and may be observed by looking at that mirror. Initially, the fringes will probably be very fine and a very small movement of $M_1$ will make them disappear. Adjustment of $M_2$ should also be done carefully and by adjusting both a little at each time it will be possible to get broad, strongly-coloured fringes into the field of view.

3.5 The optical thickness of a thin transparent film.

Carefully adjust $M_1$ so that the central dark fringe is in the centre of the field of view. A reference marker may be placed close to $M_1$ to help define the fringe position. Place the thin Mylar film in one arm of the interferometer and note the shift in $M_1$ necessary to bring the dark fringe back to its original position. Repeat this measurement with the film in the other arm of the interferometer. Calculate the additional optical path introduced by the film and, given that that the refractive index of Mylar is 1.64, work out the film thickness. Repeat the measurement a few times to allow for an uncertainty calculation to be made.

[Insertion of Mylar into a beam path increases the optical path light must travel, but remember, when doing this, that the Mylar is replacing air which was originally in the beam path. This needs to be considered when doing the calculation.]
In making these measurements you may notice that the central fringe becomes tinted when the film is in one arm of the interferometer. This is because the refractive index of the film varies with wavelength slightly so that its optical path is not identical for all wavelengths.

References

R.S. Longhurst, “Geometrical and Physical Optics”, pp 144-149.