UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS2112**

ASSESSMENT PATTERN : **PHAS2112A**

MODULE NAME : **Astrophysical Processes: Nebulae to Stars**

DATE : **08-May-09**

TIME : **14:30**

TIME ALLOWED : **2 Hours 30 Minutes**

2008/09-PHAS2112A-001-EXAM-35

Answer ALL SIX questions from Section A, and any THREE questions from Section B

Numbers in square brackets in the right-hand margin indicate a provisional allocation of maximum possible marks for different parts of each question.

The following may be assumed, if required:

Speed of light in vacuo: $c = 2.998 \times 10^8$ m s^{-1}
Planck constant: $h = 6.626 \times 10^{-34}$ J s
Solar mass: $M_\odot = 1.99 \times 10^{30}$ kg
Parsec: pc $= 3.086 \times 10^{16}$ m
Solar luminosity: $L_\odot = 3.86 \times 10^{26}$ W
1eV $= 1.602 \times 10^{-19}$ J
Solar radius: $R_\odot = 6.9598 \times 10^8$ m
Proton mass: 1.007276 amu $= 1.67266 \times 10^{-27}$ kg
^{4}He mass: 4.0026 amu
Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Electron scattering (Thomson) cross-section $\sigma_T = 6.65 \times 10^{-29}$ m^2
Boltzmann constant: $k = 1.381 \times 10^{-23}$ J K^{-1}
Planck function: $B_\nu(T) = \frac{2h\nu^3}{c^2}\left\{\exp\left(\frac{h\nu}{kT}\right) - 1\right\}^{-1}$; $\exp(x) \simeq 1 + x$ for $x \ll 1$

Section A
(Answer ALL questions from this section)

1. Define the specific intensity, I_ν, and [4]
   the n^{th} moment, of a radiation field. [3]

2. Summarize the most important physical processes involved in heating and in cooling of neutral clouds in the diffuse interstellar medium. (Detailed discussions and formulae are not required.) [7]

3. Sketch the normalized interstellar extinction curve. [4]
   By reference to your sketch, define and explain R, the ratio of total to selective extinction. [3]

4. Explain the conditions that prevail under Local Thermodynamic Equilibrium (LTE). [2]
   List the relevant distributions appropriate to those conditions (by name; formulae are not required). [4]

5. What is a 'Strömgren Sphere'? [2]
   Derive an expression for the radius of a Strömgren Sphere. [4]

6. Briefly outline the physical processes leading to Type Ia and to Type II supernovae. [5]
   Hence explain why the former can make useful 'standard candles' while the latter do not. [2]

Section B

(Answer any THREE questions from this Section)

7. The equation of radiative transfer for free-free emission in an ionized nebula may be written as

$$\frac{dI_\nu}{ds} = j_\nu - k_\nu I_\nu.$$

Defining all quantities, and explaining any assumptions, give
(i) A general solution for I_ν in terms of τ_ν, and [6]
(ii) Approximate solutions for $\tau_\nu \ll 1$ and $\tau_\nu \gg 1$. [4]

The free-free volume opacity may be taken to be

$$k_\nu \propto \nu^{-2.1} T_e^{-1.35} n_e n_p.$$

Use this relationship, together with your approximate solutions for the optically-thick and optically-thin limits, to derive, and thence sketch, the form of the radio-frequency spectrum of free-free emission from a nebula, $I_\nu \propto \nu^\alpha$ ($\alpha \simeq -0.1, +2$; you may assume that the Rayleigh-Jeans approximation to the Planck function is valid in the radio regime). [7]

Explain how the temperature and emission measure of the nebula might be measured from its radio spectrum. [3]

8. By consideration of a star's thermal energy and gravitational potential energy, obtain the Virial Theorem in the form [11]

$$2U + \Omega = 0,$$

defining all quantities. You may assume that

$$dP = \frac{-Gm(r)\rho(r)}{r^2} \times \frac{dm}{4\pi r^2 \rho(r)}.$$

By using your derivation of the Virial Equation, obtain an expression for a lower limit to the (mass-weighted) mean temperature of a star. [7]
For main-sequence stars, $R_* \propto M_*^{0.8}$. Use this relationship to show that the mass-weighted mean stellar temperature increases with increasing mass. [2]

9. Outline the basic steps of the nuclear reactions that constitute (*i*) the PP-I process, and (*ii*) the triple-α process. [8]
Briefly summarize subsequent fusion processes, and explain why they fail to produce elements heavier than iron. [6]
Outline how such elements *are* produced. [3]
Suppose that luminosity and mass are related by $L_* \propto M_*^3$ for main-sequence stars. Estimate the main-sequence lifetime (in years) of a 1.5$M_\odot$ star, assuming that $\sim$10% of the total mass undergoes PP processing. [3]

10. Explain the 'plane parallel' approximation for stellar atmospheres. Under what circumstances is this a reasonable approximation? [3]

Derive the equation of radiative transfer for a plane-parallel stellar atmosphere in the form [8]

$$\mu \frac{\mathrm{d}I_\nu}{\tau_\nu} \qquad S$$

defining all quantities.

What is meant by the Eddington Limit for a star? Derive an expression for the Eddington Limit for a star with a fully ionized, pure hydrogen atmosphere (where electron scattering is the dominant opacity source). [6]
Using your result, calculate the Eddington limit for a 20 $M_\odot$ star (expressing your answer in solar units). [3]

11. With the aid of a diagram, explain the term 'equivalent width'. [2]

List, with brief descriptions, the principal line-broadening mechanisms in diffuse interstellar gas clouds (you may identify the associated profile shapes, but detailed formulae are not required). [5]

The optical depth in an interstellar absorption line may be written as

$$\tau(\nu) = \frac{\pi e^2}{m_e c} f \phi(\nu) N.$$

Using this result, develop an expression for the equivalent width of a *weak* interstellar absorption line in terms of the column density of absorbers. [3]

With the aid of a diagram, explain how the equivalent widths of interstellar absorption lines change with increasing column density, for lines of any strength. (Formulae are not required.) [7]

Briefly summarize the main inferences concerning abundances in the gas phase of the diffuse, neutral interstellar medium resulting from absorption-line studies. [3]

END OF PAPER