Models of Radiatively Driven Wolf-Rayet-Star Winds

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Abstract. This work reports the first fully dynamically consistent models for radiatively driven winds of Wolf-Rayet stars of spectral class WN. It is found that a correct description of hydrostatic equilibrium (including bound-free forces), a steep velocity gradient, and continuum forces (electron scattering, bound-free and free-free absorption) are needed to achieve local dynamical consistency at the wind base. The differences of WR winds to O-star winds are discussed. Performance numbers $\eta = \dot{M}v_\infty/(L_*/c)$ up to about 10 can be explained by radiation pressure alone. It is suggested that the extreme values (up to and exceeding 100) found from the modelling of observations are probably in error.

1. Introduction

One of the most puzzling aspects in Wolf-Rayet stars is the high *performance number,* $\eta = \dot{M}v_\infty/(L_*/c)$, of their winds. Hamann et al. (1995) published values from 1–160. This is surprisingly close to the theoretical upper limit

$$\eta_{\max} = \frac{2c}{v_\infty}\frac{1}{1+(v_{\rm esc}/v_\infty)^2} \simeq 200 \qquad (1)$$

which results from equating the stellar luminosity to the wind power. Although the mass-loss rate may be overestimated due to the neglect of a clumped wind structure, and the luminosity underestimated (see the recent publications of Moffat & Robert 1994; Hamann & Koesterke 1996; Schmutz 1996, 1997; Schaerer et al. 1997; Willis et al. 1997), it is clear that performance numbers well beyond unity can only be explained by radiation pressure if a very efficient multiple scattering mechanism is at work.

Lucy & Abbott (1993) showed that multiple line scattering in the wind of a model WN5 star is able to produce a globally consistent wind (i.e., power loss by photons = power gain by wind) with a performance number around 10. However, there was a deficit of radiative acceleration in their model at the wind base, raising the question whether a high mass-loss rate could be initiated at all, rather than being accelerated to a high terminal velocity.

Here we report a systematic investigation of WN-star winds (WN stars being less evolved and more extensively analyzed than WC or WO stars) within the framework of radiation-driven wind theory, where the emphasis is on producing models which are locally as well as globally dynamically consistent. After a description of the wind model and its principal assumptions and approximations the code is shown to work well for O-star models, reproducing the wind structure of the O4 If star ζ Puppis. The differences between O- and WN-star winds are described and their consequences for the radiative acceleration are discussed. Finally, a grid of dynamically consistent WN wind models is presented and discussed.

2. The Wind Model

The wind is assumed to be spherically symmetric, homogeneous, non-rotating, and without magnetic fields. The prescribed velocity field is monotonically increasing from the photosphere to the terminal velocity. The wind velocity field is smoothly joined to the photospheric structure resulting from the integration of the balance equation of momentum (including bound-free forces) and energy (determining the temperature stratification in the photosphere; see Santolaya-Rey et al. 1997).

The wind matter is in a state of statistical and radiative equilibrium; the ionization balance is calculated by setting $J = WB(T_{\rm rad})$ with a generalized dilution factor W from spherical grey theory (see Lucy & Abbott 1993) and, for Wolf-Rayet models, $T_{\rm rad} = T_{\rm e}$ for ionization edges beyond the He II edge (under the assumption of an optically thick He II continuum). This is an empirical approach to the observed strong stratification of the ionization balance which, at present, cannot be rigorously calculated due to computational complexities arising from the UV line blocking and a possible influence of the He II Lyman-α resonance line on the ionization balance (see Schmutz 1997). For O stars, $T_{\rm rad}$ is taken as $T(\tau_{\rm Ross} = 2/3)$.

The mean intensity for energies lower than the He II edge (mainly relevant for level excitation) is determined from the output of the Monte-Carlo radiation-transport calculation (Springmann 1997) which is iterated with the wind structure (ionization and excitation balance). Level populations are calculated as in Lucy & Abbott (1993), i.e., metastable levels are in LTE with respect to the ground state (the radiation temperature is taken as the excitation temperature) and normal levels are dilutedly populated.

The Monte-Carlo radiation transport was written from scratch according to the algorithm given in Abbott & Lucy (1985), Lucy & Abbott (1993), and Mazzali & Lucy (1993), and is therefore independent of the version used by Schmutz and collaborators, which is based on an early implementation by Lucy. Apart from line opacities, the code treats electron scattering and bound-free and free-free opacities of all metals (ground states only) up to $Z = 30$. Atomic data are taken from the database compiled by A. Pauldrach and M. Lennon, which presently contains 149 ionization stages with more than 2.5×10^6 lines (further details can be found in Pauldrach et al. 1998).

For a start model with fully specified stellar and wind parameters a Monte-Carlo transport is calculated. If global consistency is not achieved, the mass-loss rate has to be changed before the next iteration starts. In further iterations the velocity field can also be changed in order to reach local dynamical consistency (i.e., the sum of external forces equal the inertial term locally) as well as possible. The final output consists of the run of luminosity, $L(r)$, and the emergent flux.

Table 1. Test model for ζ Puppis. Solar abundances were adopted.

$M/M_\odot$	60	$\dot{M}$ ($M_\odot$ yr^{-1})	5×10^{-6}
$L/L_\odot$	10^6	v_∞ (km s^{-1})	2000
$R/R_\odot$	19	β	0.8
$T_{\rm eff}$	42 000 K	η	0.49
Log g (cgs)	3.66	$T_{\rm e}$(min)	16 800 K

The radiative acceleration is computed from $L(r)$:

$$g_{\rm rad}(r) = -\frac{1}{\dot{M}}\frac{dL(r)}{dr}. \tag{2}$$

3. Validation of the O-Star Model

The simple algorithm for calculating the ionization and excitation balance works astonishingly well with O stars (taking ζ Pup is as a representative example; see Table 1), as can be seen from Fig. 1, where the ionization fractions are compared to the much more detailed non-LTE models by Pauldrach (Pauldrach et al. 1994, 1998).

Figure 2 displays the emergent flux of the Monte-Carlo calculation compared to the observed flux. Although not every single feature can be reproduced we feel confident that the radiative acceleration is close to the true one in a statistical sense. The dynamical consistency is shown in Fig. 3. A β-velocity field

$$v(r) = v_\infty(1 - R_*/r)^\beta \tag{3}$$

with $\beta = 0.8$ was found to produce excellent local dynamical consistency.

4. Comparison of a WN5 and an O-Supergiant Wind Model

For comparison with the ζ Pup model we take the WN5 model from Lucy & Abbott (1993; cf. Table 2). This WN model ($\beta = 1$ as in Lucy & Abbott 1993) is globally consistent but shows the deficiency in radiative acceleration at the wind base mentioned above (Fig. 5).

According to our semiempirical description of the ionization balance, this model shows a strong stratification of ionization (Fig. 4) which is responsible for a much more efficient line driving (see below).

5. Ionization Stratification and Photon Trapping

The ionization stratification is the reason for the much higher performance of the WR wind (performance number 9 vs. 0.5 for the ζ-Pup model). In the Sobolev approximation (where the width of a line is negligible compared to the Doppler shift of a photon over a hydrodynamical length scale, so that the interaction is

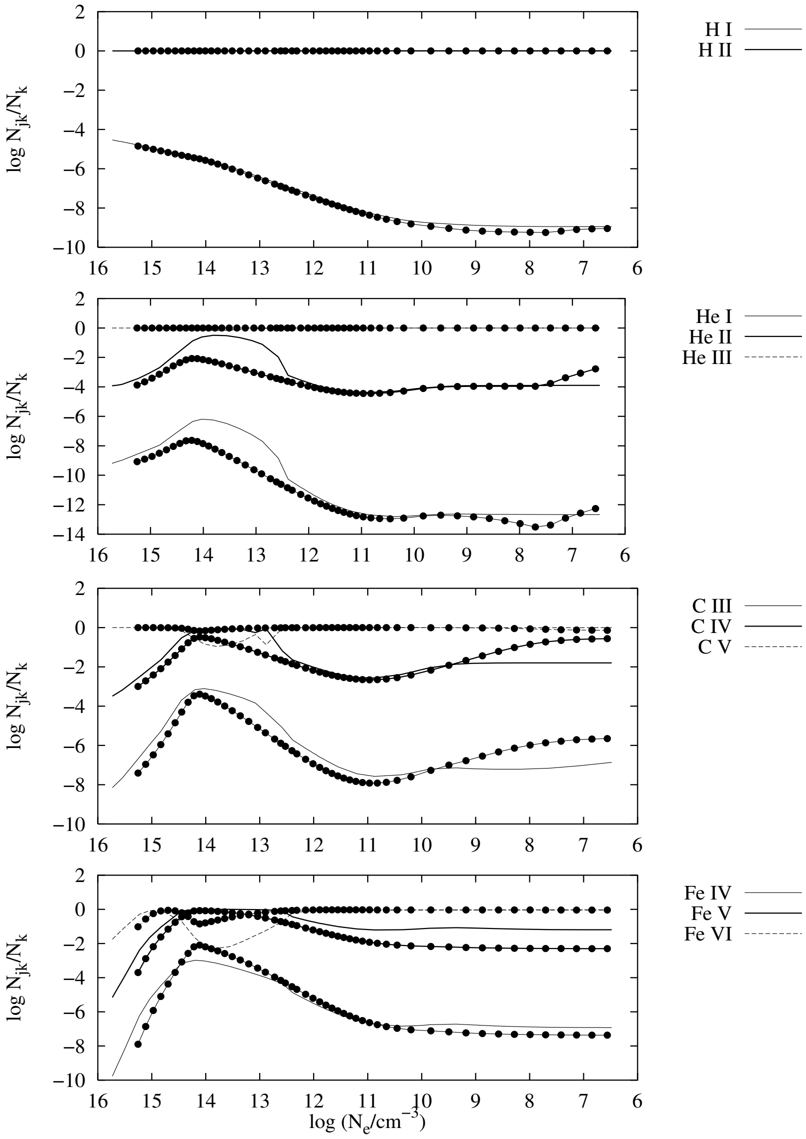

Figure 1. Comparison of calculated ionization fractions by Pauldrach (fully drawn) and our model (dots). The main ionization stages agree reasonably well for the purpose of calculating the radiation pressure.

purely local) the line acceleration is proportional to a flux-weighted sum over the absorption probabilities of the individual lines:

$$g_{\mathrm{rad}}^{\mathrm{line}}(r) = vv' \frac{L_*}{\dot{M}c^2} \sum_{\mathrm{lines}} \frac{F_{\nu_i}\nu_i}{F} \left(1 - \mathrm{e}^{-\tau_i}\right) . \tag{4}$$

This sum, which represents the effective number of lines driving the wind, is shown for both models in Figs. 6 & 7. Whereas the O-star model shows radial

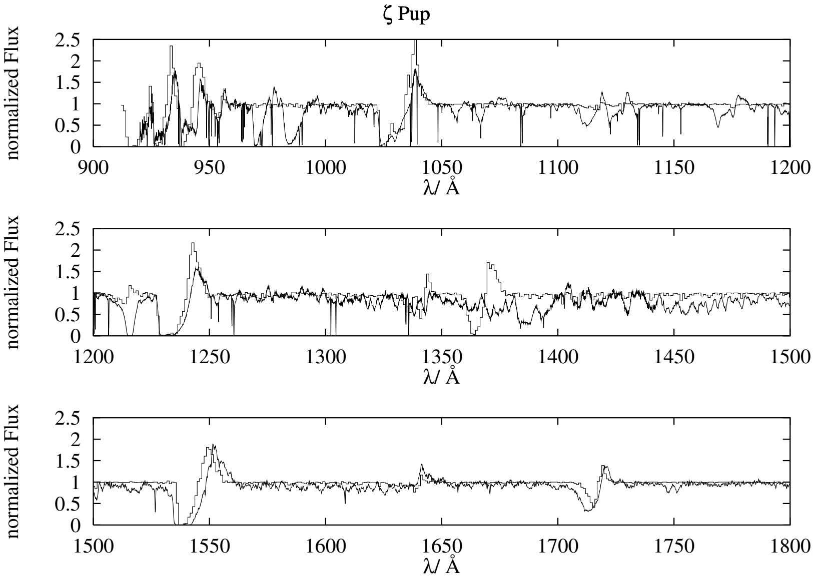

Figure 2. The UV spectrum of ζ Pup observed by IUE and Copernicus (full line; for line identifications see Pauldrach et al. 1998) and the result of the Monte-Carlo calculation (steps). Whereas minor ionization stages and their lines (N III, C III) do not agree with the observations, the contribution of the major ionization stages N IV, N V, and C IV and therefore the main contribution to the radiation pressure is reproduced reasonably well.

Table 2. Test model for generic WN5 star.

$M/M_\odot$	15.7	$\dot{M}$ ($M_\odot$ yr^{-1})	2×10^{-5}
$L/L_\odot$	2.8×10^5	v_∞ (km s^{-1})	2500
$R/R_\odot$	2.5	β	1
$T_{\rm eff}$	84 000 K	η	8.7
Log g (cgs)	4.84	$T_{\rm e}$(min)	20 000 K
Abundances (mass fractions)			
H	0.	C	3.9×10^{-4}
He	0.98	N	1.56×10^{-2}
Metals	0.02	O	3.0×10^{-4}

gaps in the summed interaction probability, there are no gaps present in the WN model. This is due to the change in ionization as shown by Fig. 8 for the case of iron: local gaps are closed at a larger radius by the emergence of a new set of lines corresponding to a new ionization stage. This is the photon-trapping effect discussed by Lucy & Abbott (1993).

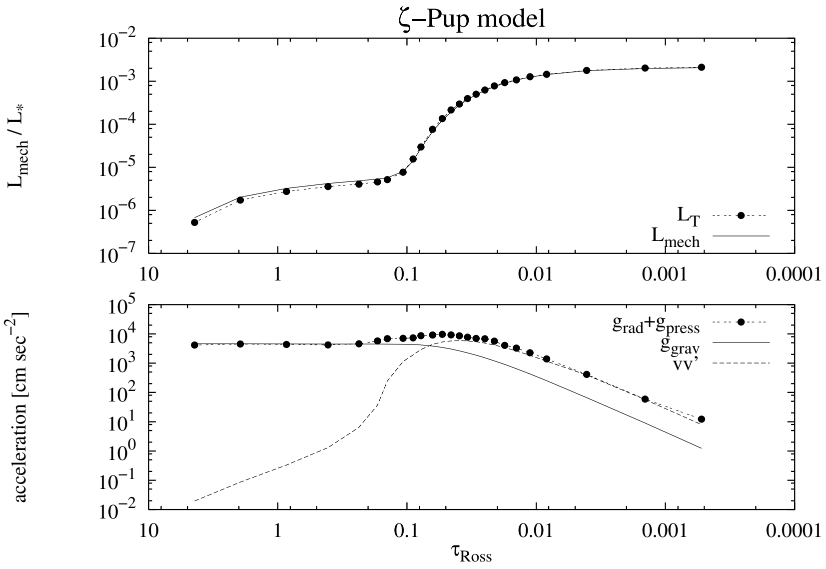

Figure 3. Upper panel: Mechanical luminosity, $L_{\rm mech}(r) = 1/2\dot{M}\,(v(r)^2 + v_{\rm esc}^2(R_*)(1 - R_*/r))$ (fully drawn), and transferred radiative luminosity, $L_{\rm T}(r)$ (dots) in units of the stellar luminosity. A model is locally consistent if both curves coincide at every point. Lower panel: Wind-driving accelerations $g_{\rm rad} + g_{\rm press}$ (dots) and decelerations $g_{\rm grav}, vv'$. For local consistency, the sum of accelerations must equal the sum of decelerations.

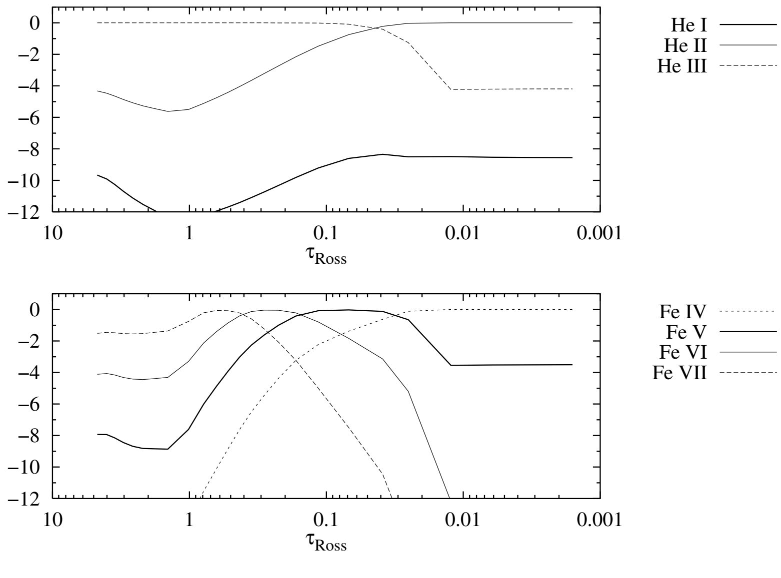

Figure 4. Ionization structure for the WN5 model.

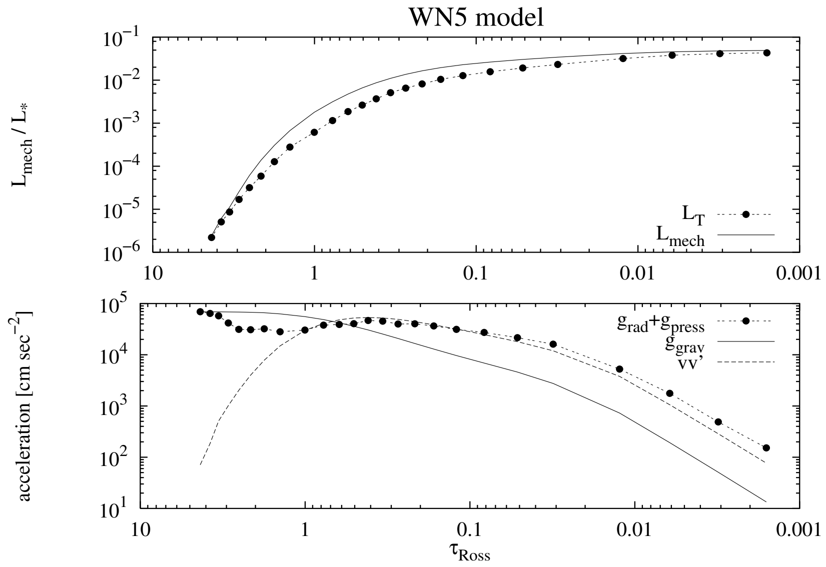

Figure 5. Dynamical behaviour of the WN model. For an explanation see Fig. 3.

6. Local Dynamical Consistency

To achieve local dynamical consistency a steeper velocity field in the lower part of the wind is required (as long as the inertial term vv' is not dominant) since over a given radial distance the photons can interact with more lines (they are doppler-shifted over a broader spectral region). The WN5 model with a generalized β-velocity field,

$$v(r) = v_\infty \left\{ (1-u)u_1^\beta + uu^{\beta_2} \right\} \tag{5}$$

(where $u(r) = 1 - R_*/r$) and values $\beta_1 = 0.6, \beta_2 = 1$ (Fig. 9) may be directly compared to the model with a simple $\beta = 1$ velocity field (Fig. 5). Also shown is the contribution of the continuum forces to the acceleration, which are important up to the region where the outflow enters the hypersonic regime.

7. A Grid of WN Wind Models

To explore the potential of radiation-driven winds in WN stars we calculated a set of models covering their putative location in the Hertzsprung-Russell diagram. Since the luminosities are questionable (cf. Section 1), a set of 4 models with different luminosities was calculated for each value of the effective temperature. The masses were taken from the theoretical mass-luminosity relation for hydrogen-free WN-stars (Langer 1989). Mass-loss rates were determined by imposing the constraints of global and local dynamical consistency.

The hottest and most luminous stars can achieve a performance number of 10. This number increases for fixed luminosity with effective temperature since

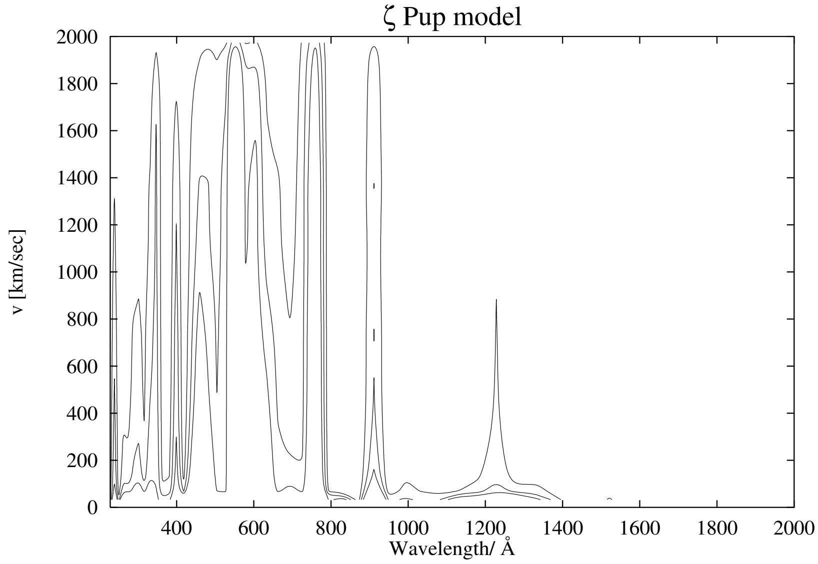

Figure 6. Effective number of wind-driving lines (the sum in Eqn. 4) in 3 contours representing the numerical values 1, 2, and 3 (the outermost contours for which scattering is certain). The constant ionization structure leads to radial gaps through which photons can escape without interaction. This limits the amount of radiative power transferred to the wind.

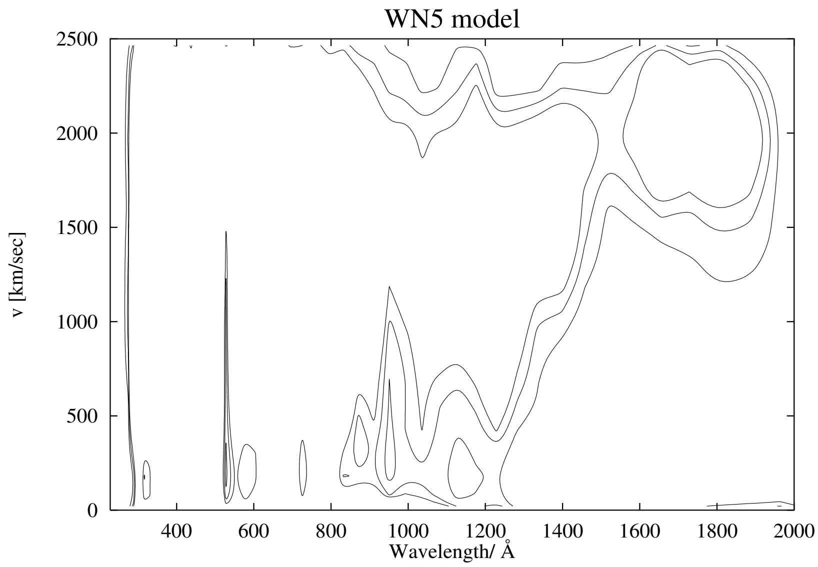

Figure 7. Same as Fig. 6 for the WN model. No gaps extending to v_∞ are present.

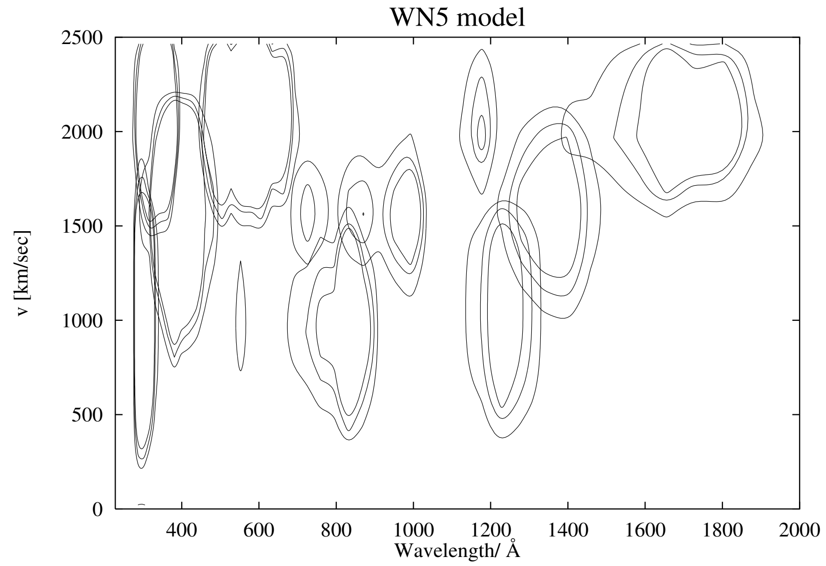

Figure 8. Effective number of lines belonging to Fe IV, Fe V, and Fe VI, centred at 1000, 1500, and 2000 km s^{-1}, respectively. Gaps in the spectral-line distribution are closed as a result of ionization shift. With increasing radius (or increasing velocity) new line ensembles become effective.

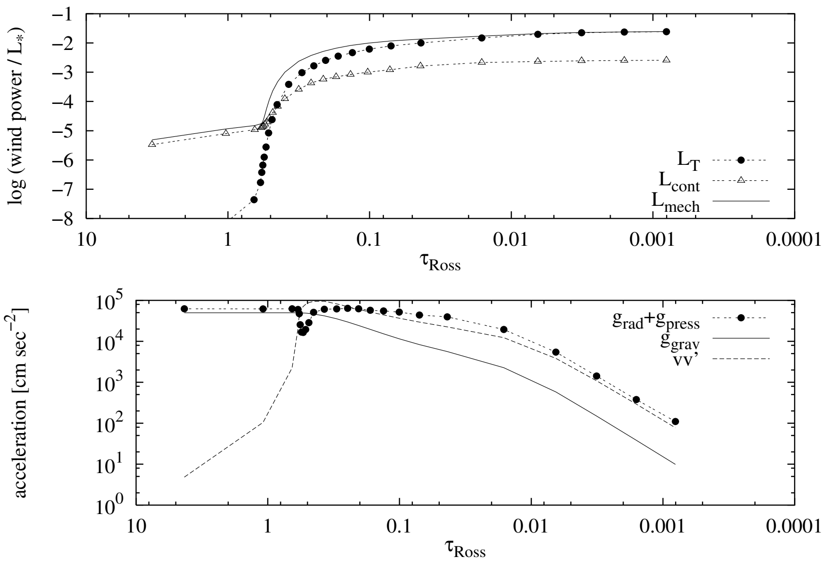

Figure 9. The WN5-model dynamics with a generalized β-field (Eqn. 5).

the stellar radius decreases, leading to larger densities and a larger contribution of continuum and line forces ('small-star effect'; Cassinelli 1991, Abbott & Conti 1987). The increase in the horizontal direction (fixed temperature, increasing luminosity) is a result of an increased continuum contribution to the acceleration (if mass-loss rate and luminosity increase in the same proportion the line force does not change).

Table 3. A grid of wind models. Table entries indicate the mass-loss rates resulting from the Monte-Carlo simulation ($\log \dot{M}/[M_\odot\ \mathrm{yr}^{-1}]$, left) and the corresponding performance numbers (right).

		$\log L_*/L_\odot$							
T_{eff} (kK)	v_∞ (km s^{-1})	5.0		5.2		5.4		5.6	
90	2300	−5.20	7.1	−5.00	7.1	−4.75	8.0	−4.45	10.1
80	2000	−5.15	6.9	−4.95	6.9	−4.70	7.8	−4.40	9.8
70	2000	−5.20	6.2	−5.00	6.2	−4.75	6.9	−4.45	8.7
60	2000	−5.30	4.9	−5.00	6.2	−4.75	6.9	−4.45	8.7
50	1800	−5.35	3.9	−5.15	3.9	−4.85	5.0	−4.55	6.2
40	1500	−5.75	1.3	−5.55	1.3	−5.10	2.3	−4.80	2.9
35	1300	−6.40	0.25	−6.15	0.28	−5.90	0.32	−5.55	0.45
30	1000	–	–	−6.30	0.16	−6.10	0.16	−5.80	0.20
Mass ($M_\odot$):		9.1		11.5		14.7		19.0	

8. Conclusions

Radiation-driven wind theory can explain performance numbers in WN winds up to about 10. A comparison of computed mass-loss rates and performance numbers with observationally inferred values by use of detailed wind models reveals that the latter values are often a factor of 10 larger. However, there is increasing evidence that mass-loss rates are overestimated and luminosities underestimated. For those cases where independent estimates of stellar and wind parameters exist there does not seem to be a contradiction between observation and predictions from wind theory (e.g., γ^2 Vel; η was previouly thought to be $\sim$110, but is now believed to be of order 10; St-Louis et al. 1988, Schaerer et al. 1997, Stevens et al. 1996, Schmutz et al. 1997).

The largest remaining problem in WR winds is an understanding of the strongly stratified ionization structure from first principles, which in our models has been modelled semiempirically.

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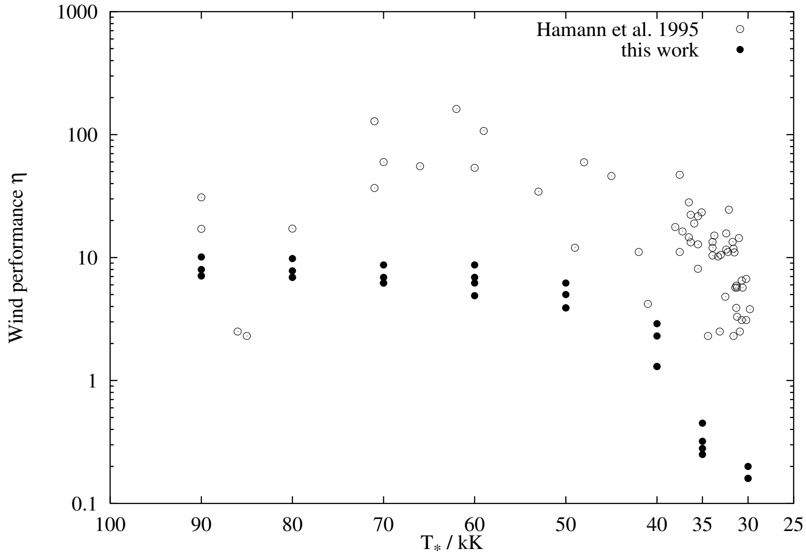

Figure 10. A comparison of computed performance numbers (filled dots) with observationally inferred values from Hamann et al. (1995, open dots). The latter values are generally considerably larger than our values. However, the mass-loss rates may be overestimated a factor of 3 due to the neglect of clumping and the luminosities may be underestimated (see Introduction). Notable exceptions are the stars WR3 and WR46 (at 85kK and 86kK); just these stars happen to be regarded as more reliably analyzed by Hamann & Koesterke (1996), since they have been analyzed using lines of nitrogen as well as helium. Hamann et al. (1996) speculate that a reanalysis with models including nitrogen would shift stars which now appear at low temperatures to higher temperatures and luminosities.

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Discussion

Langer: To what extent is the inner radius of your model atmosphere critical to your calculations? Last time we discussed this I think you had problems supporting the wind even for performance numbers of 5 or 10 when you were taking as an inner radius the stellar radius we get from structure models.
Springmann: The problem with these models was that the stellar radii were so small, and the models so hot, that I had difficulty in the transition region; to reach the low ionization stages I need low ionization temperatures in that region. With high ionization stages there is not enough line force (the lines lie predominantly in the He II continuum which is optically thick). Also, for small radii the wind has to escape from a much deeper gravitational potential, so that there is less power available for wind acceleration.
Schmutz: When I did similar Wolf-Rayet modelling, I found it impossible to produce a force around the photosphere that could support the wind. The reason is that, as long as the photons can switch their frequency, they always move from regions that are optically thick to 'windows' that are optically thin. You seem to have solved this problem, but I don't understand how; can you explain that?
Springmann: What do you mean by optically thin region? Somewhere where the lines are optically thin, or the continuum?
Schmutz: The problem is just above the photosphere, where the lines are optically thick, and the continuum is just on the turnover. The photons switch

frequency until they find 'holes' and flow out, so I always find a big drop in force around this region.
Springmann: Continuum processes help just around this transition point. Then you have Fe VI, for example, coming in very soon, and once you have that there is enough line force to support the wind.
Schmutz: Sure, as soon as you switch ionization levels there's no problem, but you can't switch too early otherwise you have a conflict with the observed spectrum. So you have a problem at the beginning, where the line driving is not yet sufficient but the continuum isn't enough.
Springmann: I had this problem at the beginning, but it went away to a large degree after I upgraded my database to Kurucz opacities, because it turned out we were lacking major line opacities from iron.
Schmutz: I was always using Kurucz opacities, so this doesn't help... maybe we need to make a detailed comparison of our models.
Conti: What about the dog that didn't bark; by which I mean, if it isn't radiation holding it up, what is? It could be the DT force (the 'Dick Thomas force', for those not in the know), but I'm not much of a believer in that. So I say, okay, we've got to performance numbers of 10; maybe that's not yet good enough, but not good enough compared to what? Answer, analyses with Wolf-Rayet spectroscopic models, and those models make other predictions of things we don't observe, so maybe part of the problem is there. A while ago, a performance number of unity was regarded as some sort of limit, and here we have 10, so perhaps we don't have too far to go. Or perhaps it *is* the DT force, I don't know... [laughter].
Kudritzki: Going back to Norbert's question; you have a problem that you don't get enough force in the situation where you're still very hot but the He II continuum is still very thick. But you make a very crucial assumption, which is that the radiation temperature is equal to the local temperature, so that you're optically thick in the continuum and the mean intensity J_ν is equal to the Planck function. By no means does this have to be the case in such a situation; the source function is normally the Planck function divided by the departure coefficient of the ground state of the corresponding ion, and the latter could deviate enormously from unity. If it is much smaller than unity you would have a much stronger radiation field and therefore a larger force.
Springmann: But I have no lines redwards of 228Å at such high temperatures; the dominant ions have all of their lines in the He II continuum.
Kudritzki: Are you sure that's not incompleteness of the line list?
Springmann: No, the line list is complete for lines redward of 228Å.