Effects of Gravity Darkening on Radiatively Driven Mass Loss from Rapidly Rotating Hot Stars

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Abstract. We investigate the effect of gravity darkening on the latitudinal variation of radiatively driven mass loss from rapidly rotating hot stars. Previous analyses have assumed a uniformly bright stellar surface and concluded that wind mass flux and density should increase with the increased centrifugal force toward the wind *equator.* In contrast, we show here that a gravity darkening in which the surface flux scales with the effective (centrifugally reduced) gravity leads to dramatically different wind morphology, with the strongest mass loss now occurring toward the relatively bright *poles.* We also review recent work that indicates nonradial (poleward) components of the line-driving force in such rotating winds can effectively inhibit the equatorward wind deflection needed to form an equatorial wind-compressed disk. Finally, we examine the equatorial bistability model, and show that a sufficiently strong jump in wind driving parameters can, in principle, overcome the effect of reduced radiative driving flux, thus still allowing moderate enhancements in density in an equatorial, bistability-zone wind.

1. Introduction

There is much observational evidence that the equatorial regions around rapidly rotating hot stars generally have an enhanced wind density, perhaps (e.g., as in Be stars) even a circumstellar disk. Previous theoretical analyses of winds from rotating hot stars have thus largely focussed on the various effects that can increase the equatorial density, including: enhanced mass flux and reduced wind speed associated with the centrifugally reduced effective gravity near the equator (Friend & Abbott 1986; Pauldrach, Puls & Kudritzki 1986); Wind Compressed Disks (Bjorkman & Cassinelli 1993; Owocki, Cranmer & Blondin 1994; Ignace, Cassinelli & Bjorkman 1996); or enhanced mass loss from a 'bi-stability zone' triggered by the reduced radiation temperature near the equator (Lamers & Pauldrach 1991).

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All these previous analyses have implicitly assumed that the radiative flux is uniformly distributed over the stellar surface. However, to the extent that radiation dominates the energy transport in the envelopes of such hot stars, the classical analysis of von Zeipel (1924) implies a 'gravity-darkening' effect, in which the radiative flux from near the equator is reduced in proportion to the centrifugally reduced effective gravity there. Here we review results from recently developed 2-D wind simulations that incorporate the effects of such gravity darkening, as well as nonradial components of the line-driving force (Owocki, Cranmer & Gayley 1996). As discussed below, both effects turn out to have a surprisingly strong impact on the latitudinal mass distribution of a stellar wind. To provide a basis for physical understanding of these unexpected results, let us first review scaling relations predicted from 1-D models of line-driven winds.

2. Scaling Laws From 1-D Models

Initial investigations (Friend & Abbott 1986; Pauldrach et al. 1986) of the possible role of rotation on radiatively driven winds derived 1-D models based on the standard line-driving formalism of Castor, Abbott & Klein (1975; hereinafter CAK), but now adding the effect of an outward centrifugal acceleration in the equatorial plane, $g_{\rm cent}(r) = v_{\rm rot}^2 R^2/r^3$, where $v_{\rm rot}$ is the equatorial rotation speed at the stellar surface radius, $r = R$. Although this centrifugal term declines faster with radius than gravity, the mass loss is fixed at a critical point quite near the stellar surface. This suggests that the effect on the local mass flux at any colatitude θ of a rotating star can be written in terms of a centrifugally reduced, *effective* surface gravity

$$g_{\rm eff}(\theta) = \frac{GM}{R^2}\left(1 - \Omega \sin^2\theta\right), \tag{1}$$

where G and M are the gravitation constant and stellar mass, and $\Omega \equiv v_{\rm rot}^2 R/GM$. We thus rewrite the standard CAK mass-loss-rate scaling law in terms of *surface* values of the mass flux $\dot{M} = \rho v$, radiative flux $F = L/4\pi R^2$, and effective gravity $g_{\rm eff}$, relative to corresponding polar ($\theta = 0$) values $\dot{M}_0$, F_0, and $g_0 = GM/R^2$,

$$\frac{\dot{M}(\theta)}{\dot{M}_0} = \left[\frac{F(\theta)}{F_0}\right]^{1/\alpha} \left[\frac{g_{\rm eff}(\theta)}{g_0}\right]^{1-1/\alpha}, \tag{2}$$

where α is the usual CAK exponent.

For example, if we take $F(\theta) = F_0$, then we obtain the scaling

$$\frac{\dot{M}(\theta)}{\dot{M}_0} = \left[1 - \Omega\sin^2\theta\right]^{1-1/\alpha} \quad ; \quad F(\theta) = F_0\,, \tag{3}$$

which, for $\sin\theta = 1$ and $\alpha \simeq 0.6$, provides a reasonably good fit to, e.g., numerical results plotted in Fig. 4 of Friend & Abbott (1986). Since $\alpha < 1$, the exponent $1 - 1/\alpha$ is negative, implying that the mass flux increases monotonically from pole ($\sin\theta \to 0$) toward the equator ($\sin\theta \to 1$). On the other hand, if we apply the standard von Zeipel (1924; see also Kippenhahn & Weigert 1990) gravity-darkening law that the surface flux itself varies in proportion to the centrifugally reduced effective gravity, $F(\theta) \sim g_{\rm eff}$, we obtain

$$\frac{\dot{M}(\theta)}{\dot{M}_0} = 1 - \Omega \sin^2\theta \quad ; \quad F(\theta) \sim g_{\rm eff}(\theta) \tag{4}$$

so that the mass flux now *decreases* towards the equator, with a maximum at the *pole* (see below)!

Friend & Abbott (1986) likewise find that the terminal wind speed is decreased by rotation in the equator. We can also approximate this result within our 1-D concept of a centrifugally reduced gravity to predict a latitudinally varying wind terminal speed that scales with an effective escape speed, $v_\infty(\theta) \sim v_{\rm esc}\sqrt{1-\Omega\sin^2\theta}$. The latitudinal variation of density is then obtained from $\rho \sim \dot{M}/v_\infty$.

3. 2-D Dynamical Simulations of Rotating Winds

3.1. Equatorial Wind-Compressed Disks

A major advance in modelling rotating hot-star winds was development of the elegantly simple 'Wind Compressed Disk' (WCD) paradigm by Bjorkman & Cassinelli (1993). They noted that, like satellites launched into earth orbit, parcels of gas gradually driven radially outward from a rapidly rotating star should remain in a tilted 'orbital plane' that brings them over the equator. As wind parcels from opposite hemispheres collide over the equator, they form a disk of compressed gas. A key simplification here is to assume that, like gravity, the radiative driving is a radially directed, *central* force, so that the total angular momentum of each individual wind fluid parcel is conserved, fixed by the rotation at its initial latitude at the wind base (i.e., at the subsonic stellar surface), and remaining in a fixed plane perpendicular to the angular-momentum vector.

To test this WCD paradigm, Owocki, Cranmer & Blondin (1994) carried out 2-D hydrodynamical simulations of line-driven winds from rotating hot stars, using the finite-disk, spherical-star form of the usual CAK formalism (Friend & Abbott 1986; Pauldrach et al. 1986) to compute the line-driving force, which is thus still taken to be purely radial. The results, shown in Fig. 1a for their standard 'S-350' model (a B2 star with $v_{\rm rot} = 350$ km s^{-1}), generally confirm the basic tenets of the WCD model, with certain detailed modifications (e.g., infall of inner disk material). The overall wind morphology consists of a relatively fast, low-density polar wind, plus a dense equatorial disk with slow outflow in its outer part. Figure 1a shows density contours for this standard WCD case, plotted vs. radius and colatitude, with superposed vectors representing the magnitude and sense of the latitudinal velocity component $v_\theta(r,\theta)$. The WCD, manifest here by the strong equatorial extension of higher-density contours, is the direct result of the flow compression associated with the equatorward sense of the latitudinal velocity from both the northern and southern hemispheres.

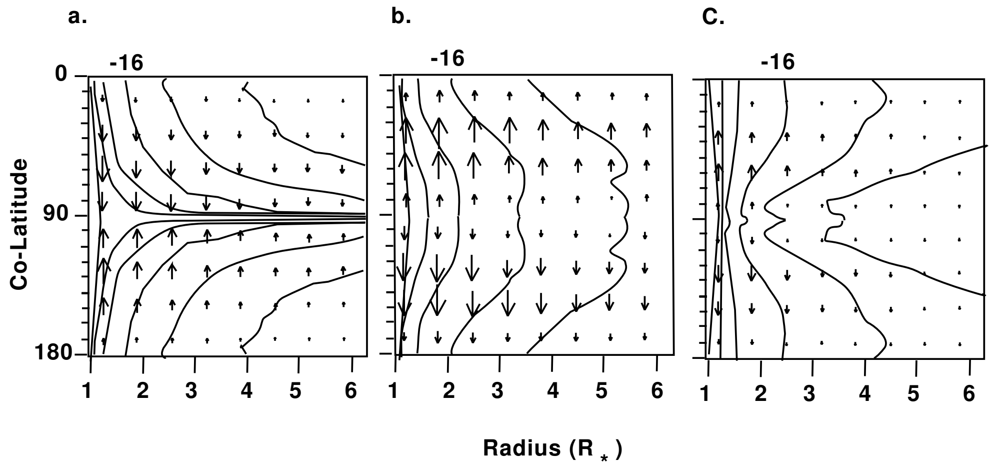

Figure 1. Contours of stellar-wind density plotted vs. colatitude θ and radius r, spaced logarithmically with two contours per decade, with label denoting the contour for $\rho = 10^{-16}$ g cm^{-}3. The superposed vectors represent the latitudinal velocity, with the maximum length corresponding to a magnitude of v_θ =100 km s^{-1}. The three panels show the cases (a) without nonradial forces or gravity darkening; (b) with nonradial forces but no gravity darkening; and (c) with both nonradial forces and gravity darkening.

3.2. Inhibition of WCD by Poleward Component of the Line-Force

For rotating stars, the line acceleration can generally also have *nonradial* components. Within the CAK formalism of a fixed ensemble of lines with a power-law number distribution in opacity, its vector form is given by integration over solid angle Ω_* of the stellar core intensity I (Cranmer & Owocki 1995),

$$\mathbf{g}^{\mathrm{rad}}(\mathbf{r}) = \frac{K}{W^{\delta}\rho(\mathbf{r})^{\alpha-\delta}c} \int_{\Omega_*} d\Omega\, \mathbf{n}\, I(\mathbf{n},\mathbf{r}) \left\{\mathbf{n}\cdot\nabla\left[\mathbf{n}\cdot\mathbf{v}(\mathbf{r})\right]\right\}^{\alpha}, \tag{5}$$

where K is proportional to the usual CAK line-normalization constant k, and the exponent δ accounts for the ionization-state dependence on density ρ and dilution factor W (Abbott 1982). Note that the weighting for the force contribution of each ray along a direction $\mathbf{n}$ is proportional to the projected velocity gradient in that direction, $\mathbf{n}\cdot\nabla\left[\mathbf{n}\cdot\mathbf{v}(\mathbf{r})\right]$.

Owocki, Cranmer & Gayley (1996) carried out simulations of rotating winds including these nonradial line-force components. Figure 1b shows the corresponding wind density structure for the S-350 model, still assuming a uniformly bright stellar core. As predicted in the above 1-D analysis, the reduced gravity, enhanced mass loss, and lower flow speed near the equator yield a broad, moderate density enhancement in the equatorial wind. But there is *no wind-compressed disk.* Indeed, the sense of the superposed vectors is now reversed, indicating that the latitudinal velocity is now *away from the equator.* As such, there is no longer any wind compression effect, and so the tendency to form a WCD is completely inhibited.

This latitudinal flow reversal is a direct consequence of a *poleward* component of the line-force, which arises primarily from asymmetries in the line-of-sight velocity gradient, operating through the velocity-gradient weighting of the angle integral in eqn. (5). The lower effective gravity near the equator implies generally lower outflow speeds there, and thus, from most mid-latitude locations in the wind, the line-of-sight velocity gradient is stronger when looking toward the equator than toward the pole. Hence photons from near the equator impart a stronger impulse than those from near the pole, enhancing the net poleward component of the line force. The magnitude of this poleward force is small, generally not much more than 10% of the radial line force; but the equatorward flow speeds are similarly small, i.e., less than 100 km s^{-1} in the WCD model, or only a few per cent of the maximum radial speed. Thus, while the radial line force must be strong enough to overcome the stellar gravity to drive an outflow to terminal speeds of more than 1000 km s^{-1}, the poleward latitudinal line force is unopposed by any other body force, and need only overcome inertial terms characterized by a modest, $<$ 100 km s^{-1} equatorward drift. From this perspective, it thus seems clear that the derived nonradial forces should indeed be dynamically quite significant in redirecting the equatorward drift needed for a WCD.

Finally, although the 2-D models here have an assumed azimuthal symmetry, there is nonetheless also a nonzero *azimuthal* component of the line-force, which again results from asymmetries in the line-of-sight velocity gradient. Because wind rotation speed declines with increasing radius, the velocity gradient toward the receding stellar hemisphere is greater than that toward the approaching hemisphere. Through eqn. (5), this now implies a net line force *against* the sense of rotation (Grinin 1978). Its peak magnitude is roughly comparable to that for the poleward line force, but this is sufficient to cause a modest wind spin-down, characterized by about a 20% decrease in the specific angular momentum of the equatorial wind outflow beyond a few tenths of a stellar radius from the surface.

3.3. Inclusion of Gravity Darkening

Figure 1c shows results for the corresponding model with both nonradial forces and a gravity-darkened surface flux $F(\theta) \sim g_{\rm eff}(\theta)$ (von Zeipel 1924; see also Cranmer & Owocki 1995). In this case, not only is there no disk, but the overall density in the equatorial regions is actually *reduced* relative to that at higher latitudes. This picture is in marked contrast with previous analyses that envisioned an enhanced equatorial mass loss [e.g., eqn. (3); Friend & Abbott 1986], but it agrees well with the predictions of the gravity-darkened mass-flux scaling (4). Despite the reduced gravity near the equator, the wind mass flux there is now lower, owing to the reduced radiative flux associated with gravity darkening. The superposed vectors further show that the latitudinal velocity is again away from the equator, though with a somewhat lower magnitude than in Fig. 1b, owing to the reduced poleward force associated with the reduced radiative flux from the equator.

3.4. Equatorial Bi-Stability Zone

The above calculations have assumed fixed values of the CAK parameters (α, k, and δ), but in general these can be expected to vary with variations in, e.g., effective temperature and density. A particularly important example of this is the Lamers & Pauldrach (1991) *bi-stability* model for equatorial enhanced winds in B[e] stars. At effective temperatures near 20 000 K, the increased recombination of hydrogen tends to make a wind become optically thick in the Lyman continuum, thus dramatically altering the wind's ionization/excitation balance, and so leads to a marked shift of line-driving parameters. The effect can be most easily described in terms of a decreasing CAK exponent α, with a relatively constant line-normalization parameter $\overline{Q} \simeq 10^3$, related to the usual CAK k constant by $k = \overline{Q}^{1-\alpha}(v_{\rm th}/c)^\alpha/(1-\alpha)$ (Gayley 1995). Within CAK theory, the mass flux varies as $\dot{M} \sim \overline{Q}^{-1+1/\alpha}$, and so adopting this into the scaling formulae in §2, we find for the latitudinal variation of density $\rho \sim \dot{M}/v$,

$$\frac{\rho(\theta)}{\rho_0} = \sqrt{1 - \Omega \sin^2\theta}\left(\Gamma_{\rm e}\overline{Q}\right)^{1/\alpha - 1/\alpha_0}, \tag{6}$$

where $\Gamma_{\rm e} = \kappa_{\rm e} F/g_{\rm eff} c$ is the surface Eddington factor, which is independent of latitude in the standard von Zeipel (1924) scaling $F \sim g_{\rm eff}$ for gravity darkening. The exponent α is now assumed to vary in latitude, e.g., from a typical O-star value $\alpha_0 \simeq 2/3$ at the relatively high-temperature pole, to a lower, B-star value $\alpha \simeq 1/2$ at lower latitudes, where the rotation brings the effective temperature near or below the critical bi-stability temperature of $\sim$ 20,000 K. The expression here includes the reduced driving from the lower equatorial brightness, an effect that actually was overlooked in the original Lamers & Pauldrach (1991) analysis [cf. their eqn. (4) and our eqns. (3) & (4)]. For stars with Eddington factors $\Gamma_{\rm e}$ not much less than unity, the large intrinsic value of $\overline{Q}$ implies that this shift in α can cause a strong increase in mass flux, as signified by the second term in eqn. (6). For example, for the case $\Gamma_{\rm e} = 0.5$, $\overline{Q} = 10^3$, $\alpha_0 = 2/3$, and $\alpha = 1/2$, this bistability density jump represents a quite large factor, $\sim \sqrt{500} \simeq 22$. In principle, this could occur quite abruptly near and below the latitude of the critical temperature, and thus overwhelm the more gradual tendency for the mass flux to decline with the decreasing radiative flux near the equator. As such, this model remains a viable possibility for explaining moderate increases in equatorial density, such as inferred for B[e] stars.

4. Concluding Remarks

The above results on nonradial forces and gravity darkening are both a surprise, in that they essentially reverse previous theoretical expectations, and a puzzle, in that they apparently contradict observational evidendence for enhanced densities in the equatorial regions around rapidly rotating hot stars. As regards gravity darkening, if convection were to dominate energy transport in the envelope of rapidly rotating stars, then the equatorial darkening could be substantially reduced, or even eliminated (Zhou & Leung 1990). Given the formidable difficulties of multidimensional modelling of convection in rapidly rotating stellar envelopes, this emphasizes the importance of determining reliable observational diagnostics of gravity darkening in such stars (e.g., Howarth & Reid 1993).

As regards WCD inhibition by nonradial line-forces, we emphasize the following points. First, the WCD inhibition described above is specific to *line-driven* winds, and does *not* imply a failure or weakness of the general WCD paradigm as potentially applied to outflows driven by any other mechanism. However, insofar as winds from O and B stars have traditionally, and quite successfully, been modelled using the CAK, line-driving formalism, this does represent a serious challenge for applying the WCD paradigm toward, e.g., Be stars, B[e] stars, and selected O stars (e.g., HD 93521: Howarth & Reid 1993; Bjorkman et al. 1994) inferred to have enhanced equatorial densities and/or disks. In the context of the standard CAK/Sobolev formalism, the inhibition effect seems quite robust, arising directly from the tendencies for the line-force to scale with the line-of-sight velocity gradient, and for the wind from a rotating star to be slowest at lower, near-equatorial latitudes. However, there are questions regarding its applicability to lower-density B-star winds (see, e.g., Cassinelli & Ignace 1997.) One particularly serious general question (emphasized to us by J. Bjorkman, personal communication) regards the neglect here of the strong line-driven flow instability, which ultimately should break the wind up into multiple dense blobs that might not receive the same poleward driving force; this question must be addressed in future work, e.g., within 2-D simulations using the nonlocal escape integral forms for line-driving (Owocki & Puls 1996). On the other hand, perhaps these findings can be interpreted as indicating that other mechanisms, e.g., decretion, magnetic loops, or bistability zones, should be examined as ways to produce the observed disks or equatorial density enhancements inferred in Be and B[e] stars.

Whatever the ultimate resolution to these puzzles, we hope that the results here will spur a reexamination and questioning that ultimately leads to a deeper understanding of the nature of hot-star mass loss. It is humbling that, more than two decades after the introduction of the line-driving mechanism in landmark papers by Lucy & Solomon (1970) and CAK, we are still learning fundamentally new aspects of that intricate process.

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References

Abbott, D. C., 1982, ApJ, 259, 282

Bjorkman, J. E., Cassinelli, J. P., 1993, ApJ, 409, 429

Bjorkman, J. E., Ignace, R., Tripp, T., Cassinelli, J. P., 1994, ApJ, 435, 416

Cassinelli, J. P., Ignace, R., 1997, in Luminous Blue Variables: Massive Stars in Transition (ASP Conf. Series, Vol. 120), eds. A. Nota & H. J. G. L. M. Lamers (ASP, San Francisco), p. 166

Castor, J. I., Abbott, D. C., Klein, R. I., 1975, ApJ, 195, 157 (CAK)

Cranmer, S. R., Owocki, S. P., 1995, ApJ, 440, 308

Friend, D. B., Abbott, D. C., 1986, ApJ, 311, 701

Gayley, K. G., 1995, ApJ, 454, 410
Grinin, A., 1978, Sov. Astr., 14, 113
Howarth, I. D. Reid, A. H. N., 1993, A&A, 279, 148
Ignace, R., Cassinelli, J. P., Bjorkman, J. E., 1996, ApJ, 459, 671
Kippenhahn, R., Weigert, A., 1990, Stellar Structure (Springer, Berlin), p. 436
Lamers, H. J. G. L. M., Pauldrach, A. W., 1991, A&A, 244, L5
Lucy, L. B., Solomon, P. M., 1970, ApJ, 159, 879
Owocki, S. P., Cranmer, S. R., Blondin, J. M., 1994, ApJ, 424, 887
Owocki, S. P., Cranmer, S. R., Gayley, K. G., 1996, ApJ, 472, L115.
Owocki, S. P., Puls, J., 1996, ApJ, 462, 894
Pauldrach, A. W. A., Puls, J., Kudritzki, R.-P., 1986, A&A, 64, 86
von Zeipel, H., 1924, MNRAS, 84, 665
Zhou, D., Leung, K., 1990, ApJ, 355, 271

Discussion

Kudritzki: You could easily include the δ-dependence (that is, the density dependence) in the force multiplier; you should do this, because it might well change the results again.

Owocki: I didn't mean to say that I ignore it completely; in the simple argument I was making about the change in mass-loss rate from pole to equator I neglected it for clarity, but it's in all the numerical calculations. I normally give this talk to non-hot-star people, and it's hard enough for them to follow even the simple arguments!

Puls: I have a question concerning your explanation of equator-to-pole changes in outflow. What I think you have assumed is that the major effect is, more or less, the difference in α, the power-law index of the force multiplier, whereas your overall scaling parameter, K, you kept constant. The main idea by Adi Pauldrach, and also Lamers and maybe myself, in respect of P Cygni, where this bistability idea came from, was that the effect that due just because with different density and temperature structures you change the ionization, which means you change the effective number of driving lines, which in terms of the model turns out to be much more important.

Owocki: Well, yes, I agree with you, except that the parameter K is actually a *mixture* of α and the overall normalization, so unfortunately it varies even if you keep the total number of lines constant but vary α. The argument I'd like to make is that Ken Gayley's $\overline{Q}$ parameterization will tend to be much more constant. The key point is that the way to change the total number of driving lines most easily is to shift the slope of the distribution, α. I need to talk to you guys off-line about how true it is that this $\overline{Q}$ parameter stays constant as you move across a bistability line.