Massive-Star Models With Rotation: New Numerical Results

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Abstract. We explore the consequences for stellar models of the theory of rotational mixing presented elsewhere in the present volume by A. Maeder. We show the importance of properly accounting for the interactions between meridional circulation and shear turbulence in rotating stars. We discuss astrophysically interesting issues related to massive-star evolution: the shapes of tracks in the HR diagram, the mass-luminosity relation, lifetimes, and the evolution of the surface abundances. We conclude with some remarks regarding the effects of rotation on WR-star formation.

1. Introduction

Before exploring the effects of rotation on massive-star evolution, let us present the results of a small numerical experiment showing the tight coupling which exists between the transport processes of chemical species and those of angular momentum. First, let us suppose that angular momentum is locally conserved in radiative zones. That means that the rotational velocity of each shell will decrease when the star expands, and increase when it contracts. Local conservation of angular momentum may result from a balance between the outward flux of angular momentum, essentially driven by shear instabilities, and the inward flux, resulting from meridional circulation. Let us recall here that in most of the stellar envelope during the main sequence (MS), meridional currents rise along the rotational axis and go down along the equatorial plane (see Fig. 3 in Maeder 1998). The net effect of such a movement will be to transport angular momentum from the outer parts of the star to the inner ones. Indeed, matter rising along the rotational axis does not transport any angular momentum, while matter going down along the equator brings angular momentum towards the interior.

In Fig. 1 the evolutionary track for a $40 M_{\odot}$ stellar model, obtained by assuming local conservation of angular momentum in radiative zones, is compared with a non-rotating evolutionary track. The rotating-star model had an angular velocity equal to 43% of the critical angular velocity on the ZAMS, corresponding to a surface velocity at the equator slightly greater than 250 km s^{-1}. (The critical angular velocity is reached when the gravity at the equator is balanced by the centrifugal force.) Both rotating and non-rotating models were computed with the Ledoux criterion and semiconvection.

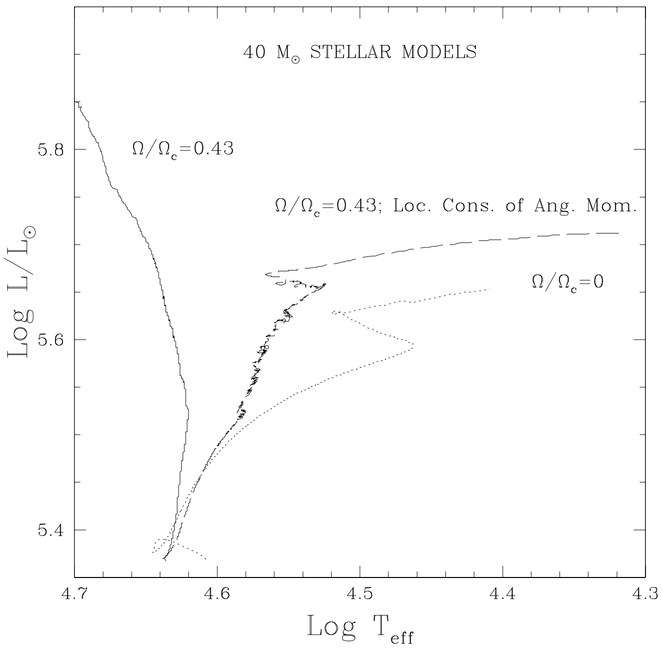

Figure 1. $40 M_\odot$ main-sequence evolutionary tracks. Dotted line, non-rotating model; dashed line, rotating model obtained by assuming local conservation of angular momentum in radiative zones; continuous line, rotating model computed without the assumption of local conservation. The curves are labelled by the adopted values of $\Omega/\Omega_{\rm crit}$, the ratios of the initial angular velocity to the critical angular velocity on the ZAMS.

From Fig. 1, one can see that the track of the rotating model differs slightly from the non-rotating one. At the red turn-off (points with the lowest effective temperature on the MS) the He surface abundance has reached the value 0.40 by mass fraction, which means an increase of 33% with respect to its initial value of 0.30. Therefore, in case of local conservation, even with a relatively high initial angular velocity, the mixing, at least in terms of surface enrichments, is moderate.

One can wonder, of course, if the hypothesis of local conservation is physically meaningful. In our model, local conservation imposes some specific values of the meridional velocity, of the order of a few 10^{-3} cm s^{-1}. This value is much lower, by about two orders of magnitude, than the value obtained by dividing the radius of the star by the Eddington-Sweet timescale, which is the appropriate timescale for large-scale redistribution of angular momentum by meridional circulation (see Tassoul 1978). As we shall see below, in more realistic stellar models local conservation is not realised, and the meridional circulation velocity is much greater than the one required to balance the effects of shear turbulence.

Let us check this point by computing a stellar model without the hypothesis of local conservation of angular momentum. In this case the angular velocity of a shell inside the star will change due to the concomitant effects of four different processes, the first two being already explicitely accounted for in the 'local conservation' model, namely:

1. the changes of angular velocity due to changes of radii caused by contraction or dilatation of the shell, and

2. the changes brought about by convective mixing.

Two more are now explicitely taken into account without any *a priori* hypothesis of their mutual effects:

3. changes due to meridional currents, and

4. changes due to shear turbulence.

In such models, the velocities of the meridional currents are two orders of magnitude greater than in the local-conservation model, implying a net inward flux of angular momentum. This can be seen in Fig. 2, where the profiles of the angular velocity inside the local-conservation model and in the model with no local conservation are compared. The profiles shown here correspond to the same evolutionary stage, i.e., to the same value of the central hydrogen content, in this case 0.35 by mass fraction. The shear tends to weaken any gradient of Ω. Instead, the meridional circulation, as seen above, tends to reinforce the gradient of Ω. Since this last effect is the dominant one, the net effect is an enhancement of the Ω gradient with respect to the local-conservation case.

What are the consequences on the transport of the chemical species? The transport of chemical species will be more efficient the more pronounced is the angular-velocity gradient (see Maeder 1998). Since the meridional currents reinforce the angular-velocity gradient, one obtains more mixing. And indeed, in Fig. 1 we see that the evolution of the star is greatly changed compared to the local-conservation case (compare the continuous track with the dashed one). The evolutionary track remains in the blue as a consequence of the strong mixing processes induced by rotation (see Section 2, below).

This numerical example illustrates the interactions existing between meridional circulation and turbulence in rotating stars. More precisely, meridional currents affect the transport of the chemical species both directly, by advecting matter of a given chemical composition into a zone with another chemical composition, and indirectly, by enhancing the angular-momentum gradient and thus the shear turbulence. These two effects are both important and must be taken into account explicitly.

In the remainder of this paper, we will present theoretical results obtained from stellar models including all the physical processes described in the paper by Maeder (1998).

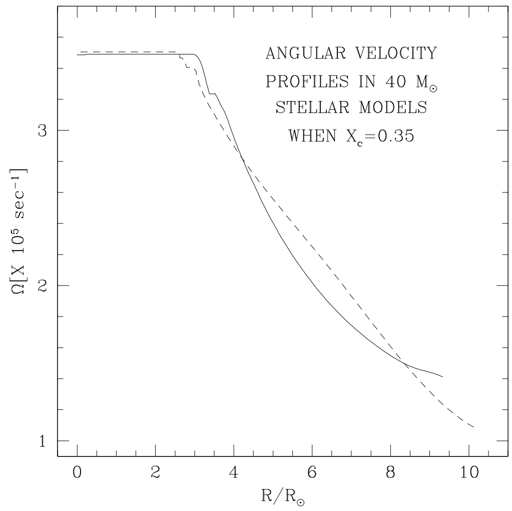

Figure 2. Angular velocity as a function of the radius inside $40 M_\odot$ stellar models when the hydrogen mass fraction at the centre, X_c, is 0.35. Dashed line, model computed assuming local conservation of angular momentum in radiative shells; continuous line, no local conservation imposed. The initial angular velocity amounts in both cases to 43% of the critical angular velocity.

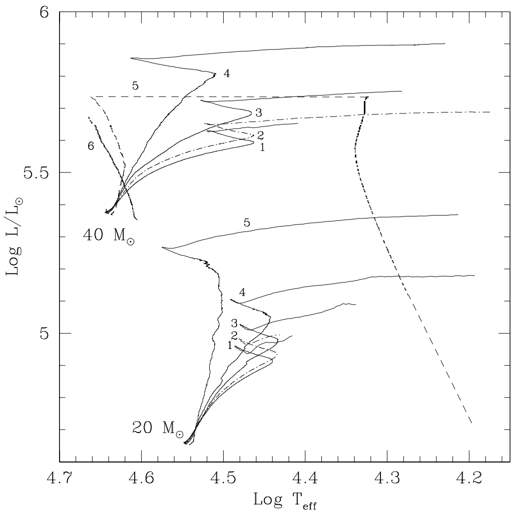

Figure 3. 40 and $20 M_\odot$ MS evolutionary tracks computed with different initial rotational velocities: track 1, $\Omega/\Omega_{\rm crit} = 0$; 2, 0.09; 3, 0.18; 4, 0.28; 5, 0.43; and 6, 0.95. Track 5 from the $40 M_\odot$ model already enters the WR phase during the MS; its redward shift at $\log(L/L_\odot) \sim 5.74$ is due to the appearence of thick WR stellar winds. Only the beginning of the MS evolution of track 6 is plotted on the figure.

2. HR Diagram

Figure 3 presents evolutionary tracks of 40 and $20M_{\odot}$ models computed with different initial angular velocities. One sees that, when the initial angular velocity increases, the tracks become overluminous with respect to their non-rotating counterparts. Moreover, they are shifted towards the blue. This results mainly from two causes. On one hand, rotational mixing erodes the molecular-weight gradient at the outer edge of the convective core. This mass enhancement of the convective core tends to increase the luminosity and to enlarge the width of the MS. On the other hand, rotation brings hydrogen-burning products such as helium to the surface (see Fig. 6). The more rapidly the star is rotating, the more efficient are the mixing mechanisms and greater is the surface helium abundance at a given point along the main sequence. This helium enhancement at the surface decreases the opacity and tends to make the star more luminous and bluer.

When the angular velocity becomes greater than a critical value, the star follows the track of an homogeneous star and always remains in the blue (see Maeder 1987). For our $40M_{\odot}$ stellar model, such homogeneous evolution occurs for initial equatorial velocities at the equator greater than about 250 km s^{-1}. When the initial mass decreases, then for the same initial fraction of the critical angular velocity, less mixing occurs (compare, for instance, the 43% evolutionary tracks for the 20 and $40M_{\odot}$ stellar models). This results from the fact that when the initial mass decreases, the ratio of the mixing timescale to the main-sequence lifetime, $\tau_{\rm mix}/\tau_{\rm MS}$, becomes greater (see Maeder 1998).

3. Mass–Luminosity Relation and Main-Sequence Lifetimes

The widening of the MS in luminosity when the initial angular velocity increases is shown in Fig. 4. One sees that in the mass range between 20 and $40M_{\odot}$, the luminosity at the red turn-off increases by a factor of between 2 and 2.5 compared to non-rotating stellar models when the angular velocity on the ZAMS passes from 0 to 43% of the critical angular velocity. The slope of the M–L relation increases only slightly when the angular velocity increases.

A method which is often used to determine the mass of a star is to find the evolutionary track which goes through the observed position of the star in the HR diagram. This method could, however, lead to quite erroneous results when evolutionary tracks with no rotation are used to estimate the mass of a fast-rotating star. As a numerical example, a star having $\log(L/L_{\odot}) \simeq 5.9$ and $\log T_{\rm eff} \simeq 4.5$ may correspond either to an initial $60M_{\odot}$ non-rotating model (with an actual mass equal to $50M_{\odot}$) or to an initial $40M_{\odot}$ stellar model initially rotating at 28% of the critical angular velocity (actual mass $34M_{\odot}$). In this case, the use of non-rotating stellar models to estimate the mass may result in an overestimate of a factor $\sim$1.5. Of course, theory predicts quite different evolutionary phase and surface abundances in both cases. Indeed, the non-rotating model presents no surface helium enrichment, and is still on the MS. By contrast, the rotating model presents a large helium enrichment (mass fraction at the surface = 0.57 compared to 0.30 initially) and is in a post-MS phase, just beginning to cross the HR diagram from blue to the red.

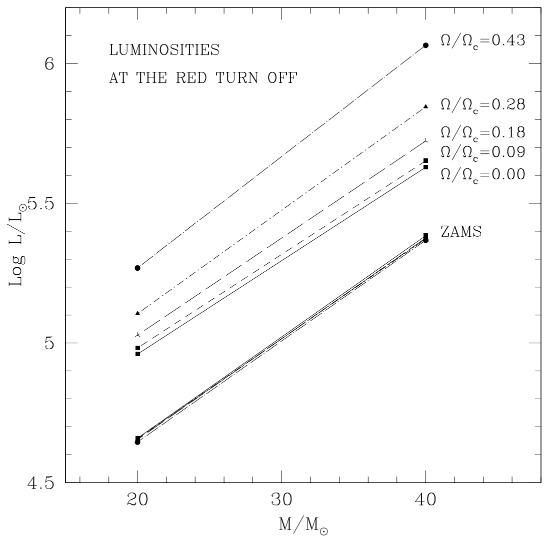

Figure 4. Luminosity on the ZAMS and at the red turn-off as a function of the initial mass for various initial values of $\Omega/\Omega_{\rm crit}$.

Figure 5 shows the increase of the MS lifetime with increasing initial angular velocity. The mass enhancement of the reservoir of the nuclear fuel (see Section 2) overcomes the effect of enhanced luminosities (see Fig. 4) on the duration of the H-burning phase. The net result is thus an increase of the MS lifetime when the angular velocity increases. For an angular velocity equal to 30% of the critical angular velocity, the increase amounts to about 35% for both the 20 and $40M_\odot$ models.

4. Evolution of Surface Abundances

The surface helium abundance ($Y_{\rm s}$) is plotted in Fig. 6 as a function of the central hydrogen content ($X_{\rm c}$) by mass fraction. As evolution proceeds, the central hydrogen content passes from its initial value, equal to 0.68, to zero. If there is no surface enrichment during the MS, the evolutionary track in this plane is a horizontal line, fixed at an ordinate equal to 0.30, i.e., the initial helium abundance. If, on the contrary, the star is evolving homogeneously, the track follows a straight line given by the equation $Y_{\rm s} = 1. - X_{\rm c} - Z$. When the angular velocity varies from zero to 95% of the critical angular velocity, different evolutionary tracks, intermediate between these two extreme cases, are followed.

We see that the greater the angular velocity, the earlier will there be helium enrichment at the surface. For a $40M_\odot$ stellar model, for $\Omega/\Omega_{\rm crit}$ greater than 20%, surface helium mass fractions greater than 0.4 are already reached on the MS. When the initial mass decreases, then for a given fraction of the critical angular velocity on the ZAMS, and for the same value of the central hydrogen content, the surface helium enrichment is weaker (see Section 2).

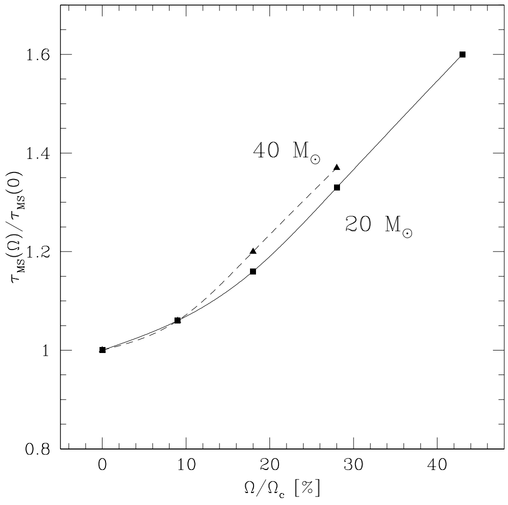

Figure 5. Main-sequence lifetimes, in units of MS lifetimes for non-rotating stellar models, as a function of $\Omega/\Omega_{\rm crit}$ on the ZAMS; $\tau(0) = 3.975 \times 10^6$ yr and 6.923×10^6 yr for 40 and $20 M_\odot$ models.

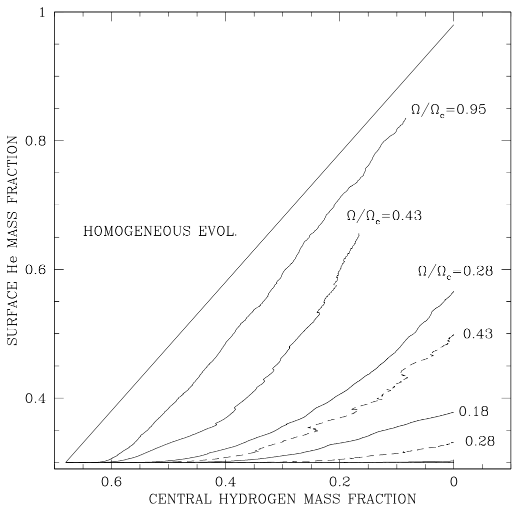

Figure 6. Evolution of the surface helium abundance as a function of central hydrogen abundance for 40 and $20 M_\odot$ stellar models. Continuous lines, $40 M_\odot$; dashed lines, $20 M_\odot$. The curves are labelled by the value of $\Omega/\Omega_{\rm crit}$ on the ZAMS. The evolutionary track followed when the star evolves homogeneously is indicated.

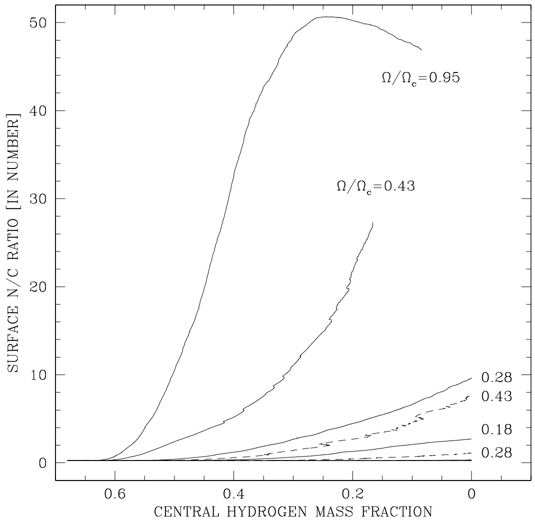

Figure 7. Evolution of the N/C surface ratio as a function of the central hydrogen abundance for 40 and $20 M_{\odot}$ stellar models. Continuous lines, $40 M_{\odot}$; dashed lines, $20 M_{\odot}$. Near each curve the adopted value of $\Omega/\Omega_{\rm crit}$ on the ZAMS is indicated. The initial N/C ratio is $\sim$0.25 in number. When the CN cycle reaches equilibrium, the ratio becomes N/C $\simeq$ 50.

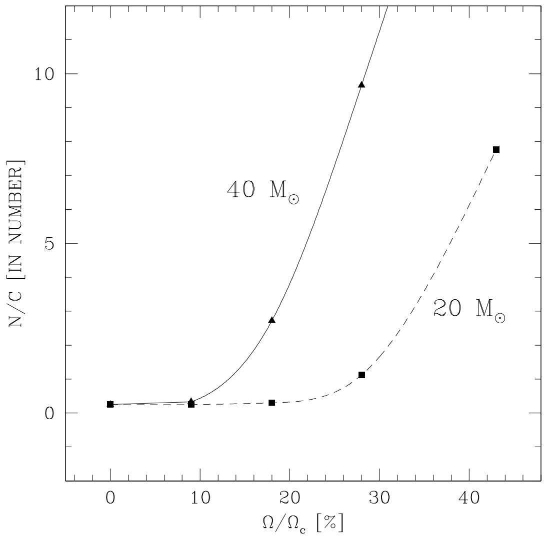

Figure 8. N/C surface ratio [by number] as a function of $\Omega/\Omega_{\rm crit}$ on the ZAMS for our 40 and $20 M_{\odot}$ stellar models during the blue-supergiant phase (see text).

Helium enrichments are accompanied by other changes. For example, the N/C ratio increases with time. The same is true for the N/O ratio, while the $^{12}C/^{13}C$ ratio is expected to decrease with time. As an example, the evolution of the N/C ratio as a function of the central hydrogen mass fraction is plotted in Fig. 7.

Figure 8 shows the N/C ratios predicted at the surface of our 40 and $20 M_\odot$ stellar models in the blue-supergiant phase for different initial angular velocities. By 'blue-supergiant phase', we mean here the phase just following the end of the MS ($\log T_{\rm eff}$ around 4.2–4.3), when the star is crossing the HR diagram from the blue to the red. We see that a $20 M_\odot$ stellar model begins to present non solar ratios for $\Omega/\Omega_{\rm crit}$ greater than 30%. Let us recall here that many observations seem to point towards the existence of N-enriched blue supergiants (see, e.g., Walborn 1976, 1988; Gies & Lambert 1992; Lennon 1994; Venn 1996). Careful comparisons will be presented elsewhere; let us just say here that rotation appears to be a very promising way to explain these non-solar surface abundances.

5. Conclusion: Rotation and WR-Star Formation

To conclude this paper, let us give some elements of response to the following question: from the above results, what can we say about the effects of rotation on WR-star formation ? We shall consider here only the single-star evolutionary scenario, although mixing processes similar to those induced by rotation might also occur in tight binary systems as a result of tidal effects.

Mixing induced by rotation favours the early appearence of H-burning products at the surface, and therefore makes WR-star formation easier. This means that, in particular, the lower mass limit for WR formation will decrease when the angular velocity increases, and that entry into the WR phase will occur at earlier and earlier stages when the star rotates more and more rapidly. In this respect, an increase of the rotation velocity acts in a similar way to an increase of the mass-loss rates by stellar winds.

For instance, The present models predict that $40 M_\odot$ stellar models with an initial angular velocity on the ZAMS greater than about 30% of the critical velocity will already become WR stars during the MS. For $20 M_\odot$ stellar models, such a situation occurs only for initial velocities greater than about 50% of the critical angular velocity. To obtain these estimates we supposed that the star enters the WR phase when it is hotter than 10 000 K and the surface hydrogen abundance by mass fraction is less than 0.4.

While rotation and mass loss may act in a similar way as far as the process of WR star formation is concerned, as said above, mixing induced by rotation deeply affects the chemical structure of the central regions of the star, contrary to mass loss. Indeed, from Fig. 9, one can see that rotational mixing smooths the hydrogen profiles obtained at the end of the MS. Moreover, hydrogen depletion extends over a greater and greater portion of the star as rotation increases. Future computations will permit an exploration of the consequences of such changes on many aspects of post-MS massive-star evolution. As an example, some preliminary comparisons confirm that rotation is a promising process for

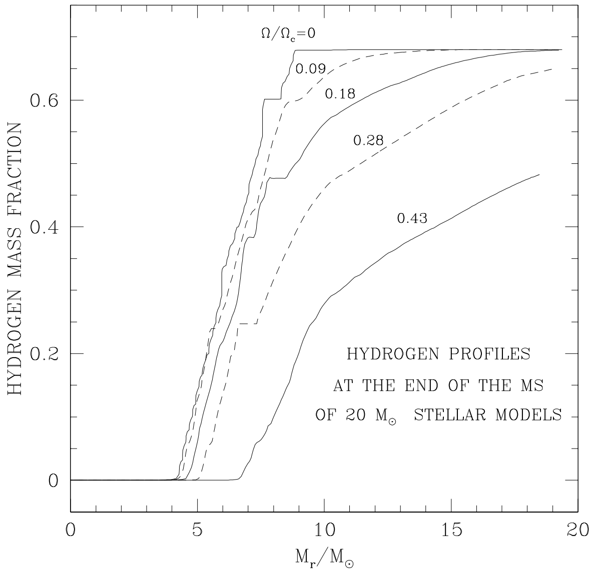

Figure 9. Hydrogen mass fraction as a function of the lagrangian mass at the end of the main sequence for $20 M_\odot$ stellar models computed with different angular velocities. Curves are labelled by the adopted value of $\Omega/\Omega_{\rm crit}$ on the ZAMS.

explaining the existence of low-luminosity, hydrogen-rich WN stars (Maeder 1995).

Finally, let us emphasize that hydrodynamical processes induced by rotation are quite complex. The models presented here are an attempt to go a little further in the physical and numerical treatment of some of them (see also Talon et al. 1997). But of course, still more work is needed for a better hydrodynamical description of turbulence in rotating stars.

Acknowledgments. This work results from a close collaboration with André Maeder during the last two years. I express my deep thanks to him for having invited me to join this wonderful adventure.

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Discussion

Kudritzki: I wanted to draw attention to a paper by Henny Lamers, with me and a couple of other authors, in A&A [325, L25, 1997]. From HIPPARCOS data, we find slow rotators on the main sequence are fainter than rapid rotators. It would be extremely interesting to see if you could explain this quantitatively.
Meynet: You mean on the zero-age main sequence?
Kudritzki: No, not *zero*-age main sequence; but stars classified as dwarfs.
Meynet: Qualitatively, I'd expect fast-rotating stars to have higher luminosities than slow-rotating ones.
Howarth: I believe I understood your mechanism for moving angular momentum about: the circulation first transports material from the core to the surface along the polar axis; the material then moves across the surface to the equator, picking up angular momentum along the way, and finally transports that momentum inwards along the equatorial plane.
Meynet: That's right.
Howarth: So that carries angular momentum from the outside in... but I understood Norbert to say that, in his picture, as the core shrank and the outside got bigger, angular momentum was transported *outwards,* to maintain the rotation rate. I don't understand how these two things fit together.
Meynet: You have two mechanisms: one which tends to reduce the radial gradient in angular velocity, and another which, most of the time, tends to strengthen it. Only models expressly including both can say whether we have an equilibrium situation or not, and which mechanism will overcome the other. In our models, the effect of meridional currents overcomes the effect of shear turbulence.
Howarth: Is that something specific to these models, or something we can expect to be generally true?
Meynet: I think that if the physics we've included is right, it must be like this. There is no fine-tuning; we've just put in the equations, and we look at what happens.
Howarth: Does Norbert want to comment?
Langer: Well, I've included both processes as well, and get something rather close to rigid rotation... we have to check to see what the differences are.
Meynet: I don't know if you [Langer] do this, but one difference could be that many people include meridional circulation through a diffusion equation, and my experience is that you can't model it well in this way; the diffusion approach always tend to smooth any gradients, and suppresses any mechanism which steepens the angular-momentum gradients. But of course we will have to examine this.

Conti: I'm very excited to hear about a new Wolf-Rayet channel being opened, although obviously there are other things going on at the same time and you've got a lot of work to do. What I was wondering about is what role would magnetic fields play – of course, they haven't yet been discovered [laughter]... but if this channel is correct, there can't be any magnetic fields, because stars will obviously brake themselves very quickly with a stellar wind *and* a magnetic field.

Meynet: I think maybe this is for the next generation!

Crowther: I just wanted to comment on the assumed main-sequence mass-loss rates of these very massive stars. I think you're using mass-loss rates based on the de Jager prescription for rates across the HR diagram, and it seems that if you look in detail at the very massive stars then you find that the factor of 2 enhancement over the de Jager rates that you assume actually gives much better agreement with the momentum relationship.

Meynet: Yes, we believe that the enhanced rates are better in the sense that, with them, our models reproduce many observations; so we provide an independent confirmation of these higher rates.