The Physics of Rotational Mixing in Massive Stars

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Abstract. The physics of the new Geneva models of stellar evolution with rotation is briefly presented. Hydrostatic effects are treated with the Kippenhahn & Thomas method modified for shellular rotation. Surface effects of rotation (i.e., distortion, changes of gravity, T_{eff}, and mass-loss rates) are also accounted for. Potentially the most important effects of rotation are those due to chemical mixing and transport of angular momentum. We treat the shear mixing explicitly with a new method and we also follow meridional circulation paying great attention to the effect of molecular-weight gradients and horizontal turbulence. The numerical methods to be applied require great care, because the system of equations describing the meridional circulation and the conservation of the angular momentum is of a high order.

1. Introduction

In this paper I shall present some basic physical processes considered in models of rotating stars. The first model results for massive stars will be shown in a companion paper by G. Meynet (Meynet 1998).

A dozen pieces of compelling observational evidence suggesting some additional mixing in stars of all masses, but particularly in massive stars, have been found (cf. Maeder 1995). Among the processes able to produce some mixing and dispersion in stellar properties, rotation is of course the first one to be considered. In fact, rotation has a long history! (See Tassoul 1978; Zahn 1983, 1992.)

The role of stellar rotation is generally ignored in evolution. We can distinguish the following effects:

1. Changes of hydrostatic equilibrium in the interior due to rotation.
2. Changes of surface properties: shape, T_{eff}, gravity, and mass-loss rates, $\dot{M}$.
3. Rotational mixing of chemicals and transport of angular momentum by meridional circulation, shear instabilities, turbulent diffusion, etc...

The various effects increasing the mass-loss rates and mixing can drastically influence the course of the evolution.

2. Changes of Hydrostatic Equilibrium in the Interior, and Surface Effects

An excellent method to treat the hydrostatic effects of rotation in a 1-D stellar-structure code was established by Kippenhahn & Thomas (1970). Strictly speaking, this method only applies when the angular velocity, Ω, has cylindrical symmetry and, as a matter of fact, it was improperly used by several authors. This is particularly relevant because recent work by Zahn (1992) has shown, in agreement with solar observations, that a law $\Omega = \Omega(r)$, where Ω is a function of only the distance r to the stellar centre, is likely to occur. Such a rotation law, called 'shellular rotation', results from the strong horizontal, 'meteorological' turbulence which homogenizes rotation on level surfaces. Meynet & Maeder (1997) have modified the Kippenhahn & Thomas system of equations for the case of shellular rotation. In that case the surfaces $\Omega(r)$ = constant are no longer equipotentials but are isobaric level surfaces for which a consistent and full system of structure equations can be written. On the whole, the structural effects are quite small. For a $40M_{\odot}$ star, initially at 90% of break-up velocity, the central T and ρ decrease by respectively 4% and 3%. The main-sequence lifetime is increased by only 2% and the evolutionary tracks undergo very small changes (see Fig. 2).

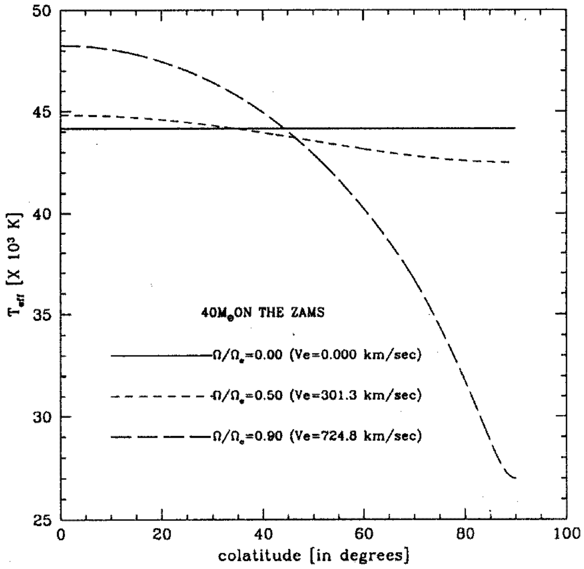

Figure 1. Local effective temperatures as a function of the colatitude at the surface of $40M_{\odot}$ ZAMS models rotating at different velocities.

At the surface of a rotating star, T_{eff} and gravity, g, are changing with colatitude ϑ (Fig. 1). The von Zeipel theorem (1924), which states that the local flux is proportional to the local gravity, is generally used to provide the local T_{eff} (but this only applies in radiative equilibrium). This means that *the spectrum of a rotating star is a composite spectrum of surface elements with varying* T_{eff} *and*

g, *seen with variable orientation angles.* Line-blanketed atmosphere models were calculated a long time ago by Maeder & Peytremann (1972) who found quite large effects of rotation on the emergent spectrum and line equivalent widths. To define an average T_{eff} for an HR diagram of rotating stars, as done in Fig. 2, we simply use $T_{\mathrm{eff}}^4 = L/(\sigma S)$, where S is the real surface of the distorted, rotating star. We must not forget, however, that complete stellar models are necessary to model the emergent spectrum properly.

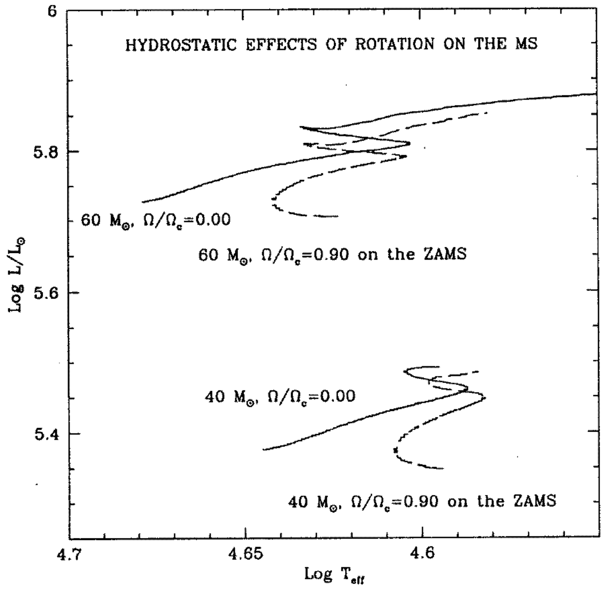

Figure 2. Evolutionary tracks for 40 and $60 M_{\odot}$ models, for $\Omega/\Omega_{\mathrm{crit}} = 0$ and $\Omega/\Omega_{\mathrm{crit}} = 0.9$.

Rotation also strongly influences the mass-loss rates of the star. The effect of the centrifugal force in radiation-driven-wind theory has been discussed by Friend & Abbott (1986). Owocki & Gayley (1997; see also Owocki, Gayley & Cranmer 1998) show that when taking into account the von Zeipel theorem one gets a local mass-loss rate $\dot{M} \sim g_{\mathrm{eff}}$, i.e., the mass-loss rate is higher at the pole than at the equator. There is, however, evidence of equatorial shell ejections in Be stars, and the amount of ejected matter may, after all, not be larger at the pole than at the equator. In particular, the dependence of opacity on T may play an important role in favouring equatorial ejection. For the time being a relation of the form

$$\frac{\dot{M}(\omega)}{\dot{M}(0)} = \left(\frac{1}{1-\omega}\right)^{\xi} \qquad \text{with} \quad \xi \simeq 0.5$$

where $\omega = v_{\mathrm{rot}}/v_{\mathrm{break-up}}$ is adopted globally for a star, in agreement with Friend & Abbott (1986; see also Langer 1997, and Owocki, private discussions). Fast rotation may thus strongly favour the mass-loss rates, with probably some important differences according to the evolutionary phases considered.

3. Rotational Mixing: The Context in Brief

For three quarters of a century rotational mixing has been studied intensively. Among the major milestones we may mention the finding of a thermal imbalance in rotating stars and the subsequent meridional circulation motions (Eddington 1926; Sweet 1950), the role of molecular-weight (μ) gradients (Mestel 1953), the existence of differential rotation and turbulence as a result of angular-momentum conservation by circulation (Kippenhahn 1973), the treatment of mixing as a diffusion process (Schatzman 1969), and studies on the role of instabilities (cf. Zahn 1983, 1992).

Zahn in particular has shown that the main instabilities producing mixing are shear instabilities, and that rotation is likely to be shellular, possibly due to strong horizontal, meteorological-like turbulence. The velocity of meridional circulation was rediscussed by Zahn for shellular rotation. Chaboyer & Zahn (1992) have shown how the transport of chemicals by circulation is reduced by the horizontal turbulence.

First grids of rotation models were made for 9, 20, 40, and 60 solar masses (cf. Meynet & Maeder 1997). They were main-sequence models including the asymptotic approximation suggested by Zahn (1992), stating that outwards transport of angular momentum by shear instabilities is just compensated by inwards transport through meridional circulation. The Richardson criterion for stability with respect to shear becomes, in the presence of μ gradients,

$$\nabla_\Omega < \nabla_\mathrm{m} \tag{1}$$

(cf. Maeder & Meynet 1996), with

$$\nabla_\Omega = \frac{1}{4}\,\frac{Hp}{g\delta}\left(\frac{dU}{dz}\right)^2 \qquad \text{and} \quad \nabla m = \frac{\varphi}{\delta}\nabla_\mu \tag{2}$$

where U is the horizontal velocity and z the vertical depth, while other symbols have their usual meaning in stellar-evolution theory. Criterion (1) means stability when the excess energy in the shear (with respect to a uniform average motion) is smaller than the energy necessary to produce a turnover of the material. The transport of chemicals by shear instabilities, when they occur, and the meridional circulation are treated as diffusion processes. The noticeable point in these models was that the μ gradients were always too strong by several orders of magnitude to allow any significant mixing. Thus, even in the case of extreme rotation, evolution underwent only small modifications. Similar results on the inhibiting role of the μ gradient were found by Chaboyer et al. (1995).

4. New Results on Shear Mixing

Since on one hand there are several pieces of evidence for more mixing in real stars (cf. Maeder 1995), and on the other the above-mentioned models do not predict any additional mixing, there is a severe problem. Thus we have to examine more thoroughly the physics of shear mixing and meridional circulation.

A disputable hypothesis in Richardson's criterion, either in the classical form or in form (1) taking into account the μ gradient, is that it is assumed that there is no other source of turbulence in the considered medium and that shear instabilities in the linear-perturbation approximation only occur when the excess energy in the shear overcomes the work necessary to produce a turnover of the material. But in rotating stars there are other sources of turbulence (semiconvection, horizontal turbulence, etc...); thus there is no threshold as that given by (1). The various existing sources of turbulence will just be fed by the excess energy of the shear, which will then further contribute to the mixing of elements. We may thus express the basic working hypothesis that *the local energy excess in the shear is degraded by turbulence, which leads to a change of the local entropy gradient* and thus of the μ and T gradients (cf. Maeder 1997). This way one finds easily that a fraction f_μ of the μ gradient can be transported by a given shear (dU/dz)

$$f_\mu = \frac{1}{4}\frac{Hp}{g\varphi\nabla_\mu}\left(\frac{dU}{dz}\right)^2 - \frac{\delta}{\varphi}\frac{(\nabla' - \nabla)}{\nabla_\mu} \tag{3}$$

where ∇' and ∇ are the internal and external gradients, respectively, and δ and φ are the usual thermodynamical coefficients. Let us now examine what happens to a fluid element having an excess $D\mu = f_\mu \nabla_\mu$ of mean molecular weight with respect to the surroundings, the fluid element having as well a temperature excess DT. The following equation applies to the temperature excess DT (see also eqn. 7 by Kato 1966):

$$\frac{\partial}{\partial t}\left(\frac{DT}{T}\right) = \frac{v_\mu}{H_\mathrm{p}}(\nabla - \nabla_\mathrm{ad}) - \frac{DT}{T\,\tau_\mathrm{adj}} \tag{4}$$

where τ_adj is the thermal-adjustment time. For a sphere with a diameter d, one has $\tau_\mathrm{adj} = d^2/(12K)$, where K is the thermal diffusivity,

$$K = \frac{4acT^3}{3\kappa\rho^2 C_\mathrm{P}}. \tag{5}$$

The equation of motion of the fluid element is

$$\frac{\partial v}{\partial t} = g\left(\delta\frac{DT}{T} - \varphi\frac{D\mu}{\mu}\right). \tag{6}$$

If $D\mu$ varies with location due to the ambient μ gradient there is no stationary solution for v_μ. However, if the Peclet number is such that

$$Pe = \frac{\tau_\mathrm{adj}}{\tau_\mathrm{dyn}} \ll 1 \tag{7}$$

then the cell rapidly adjusts its temperature to the surroundings and is almost in thermal equilibrium with it. As a consequence of the cooling, the cell moves downwards with a nearly stationary velocity so that $\frac{DT}{T} \simeq \frac{\delta}{\varphi}\frac{D\mu}{\mu}$. Thus we have

$$\frac{\partial}{\partial t}\left(\frac{DT}{T}\right) \simeq \frac{\partial}{\partial t}\left(\frac{\varphi}{\delta}\frac{\nabla_\mu}{H_\mathrm{p}}\Delta r\right) = \frac{\varphi}{\delta}\frac{\Delta\mu}{H_\mathrm{p}}v_\mu \tag{8}$$

and, from (4),

$$v_\mu = \frac{-H_\mathrm{p}}{\left(\frac{\varphi}{\delta}\nabla_\mu + \nabla_\mathrm{ad} - \nabla\right)} \frac{\varphi}{\delta} \frac{D\mu}{\mu} \frac{1}{\tau_\mathrm{adj}}. \tag{9}$$

This expression is only valid if $Pe \ll 1$, otherwise there would be no stationary velocity. Indeed, $Pe \ll 1$ corresponds to mild mixing, on a timescale longer than the Kelvin-Helmholtz timescale. This is precisely what we are interested in. We can now also apply this expression to a fluid element with a μ excess $D\mu = f_\mu \nabla_\mu$ and express the diffusion coefficient D as follows:

$$D = \frac{1}{3} \mid v_\mu \mid \Delta r. \tag{10}$$

With (3) and (9), and also considering that the ratio $\Delta r^2/d^2$ of the characteristic scales is of the order of unity, we obtain

$$D = \frac{K}{\left(\frac{\varphi}{\delta}\nabla_\mu + \nabla_\mathrm{ad} - \nabla\right)} \left[\frac{H_\mathrm{p}}{g\delta}\left(\frac{dU}{dz}\right)^2 - 4\left(\nabla' - \nabla\right)\right]. \tag{11}$$

The internal gradient ∇' is related to the adiabatic gradient by $\nabla' - \nabla = (\Gamma/\left(\Gamma + 1\right))\left(\nabla_\mathrm{ad} - \nabla\right)$, where $\Gamma = Pe/6 = v\ell/6K$. Thus the diffusion coefficient becomes

$$D = \frac{K}{N^2_\mathrm{ad}\left(\nabla_\mu\right)} \left[\alpha\left(\frac{dU}{dz}\right)^2 - \frac{4\Gamma}{\Gamma + 1} N^2_\mathrm{ad}\left(\nabla_\mu = 0\right)\right] \tag{12}$$

with the Brunt-Väisälä frequency $N^2_\mathrm{ad}\left(\nabla_\mu\right) = \frac{g\delta}{H_\mathrm{p}}\left[\frac{\varphi}{\delta}\nabla_\mu + \nabla_\mathrm{ad} - \nabla\right]$. This is the diffusion coefficient obtained when one assumes that a fraction α of the excess energy in the shear is used to transport matter. There is no threshold, as in the usual theory (cf. Zahn 1992). We note, however, that for large radiation losses, i.e., for $\Gamma \to 0$, if the fraction $\alpha = 1$ then one obtains the same coefficient D as Zahn (1992):

$$D = \frac{K}{N^2_\mathrm{ad}\left(\nabla_\mu\right)} \left(\frac{dU}{dz}\right)^2. \tag{13}$$

In the limiting case $\left(\frac{dU}{dz}\right) \to 0$ we still have

$$D = \frac{4K\left(\nabla - \nabla'\right)}{\left(\frac{\varphi}{\delta}\nabla_\mu + \nabla_\mathrm{ad} - \nabla\right)} \tag{14}$$

which, in a semiconvective zone, is similar to the coefficient obtained by Langer et al. (1983) for semiconvective diffusion. We may also take into account the effect of diffusive mixing on thermal transport (cf. Maeder 1997).

The interest of this approach is that we may avoid treating the detailed spectrum of turbulence by considering that the excess energy in the shear is transferred by the turbulent cascade to change the entropy gradient. A major change with respect to the use of Richardson's criterion is that there is no threshold to the mixing; even in the presence of a strong μ gradient, a small amount of mixing can occur.

Interestingly enough we can now understand why mixing is favoured in massive stars. Shear mixing, semiconvective mixing, and, more generally, all mixing processes resulting from dissipative effects in stars are proportional to K, the thermal diffusivity. The timescale $\tau_{\rm mix}$ for mixing is

$$\tau_{\rm mix} \simeq \frac{R^2}{D}. \tag{15}$$

With D proportional to K one has, by homology relations,

$$\tau_{\rm mix} \sim \frac{1}{M^{\zeta}}, \tag{16}$$

with $\zeta \simeq 1.7$ for electron-scattering opacities and $\zeta \simeq 3.8$ for Kramer's opacities. The main-sequence lifetimes for massive stars scale like

$$\tau_{\rm MS} = 10^{7.9} \left(\frac{M}{M_{\odot}}\right)^{-0.72} {\rm yr} \tag{17}$$

for $M \geq 15 M_{\odot}$. Comparison between eqns. (15) and (16) shows that $\tau_{\rm mix}$ decreases much faster than $\tau_{\rm MS}$ for massive stars and thus rotational mixing should be strongly favoured in massive stars. In particular, for a mass somewhere above $50 M_{\odot}$ fast-rotating stars should evolve nearly homogeneously.

5. New Results on Meridional Circulation

Meridional circulation has also been studied for a long time. Re-examination of the basic sets of equations has been undertaken and we obtained (Maeder 1998) the following improvements with respect to the usual treatment.

- The expressions are now valid for a general equation of state, and not only for perfect gases.
- The reduction of the velocity of meridional circulation by horizontal turbulence (cf. Zahn 1992) is taken into account.
- The effects of a μ gradient on the velocity of circulation are also taken into consideration. This point is certainly the most important one.

The expression found for the radial-velocity component of meridional circulation is (cf. Fig. 3)

$$U(r) = \frac{L}{M_{\star} g} \left(\frac{P}{\rho C_{\rm p} T}\right) \frac{(E_{\Omega} + E_{\mu})}{(\nabla_{\rm ad} - \nabla)\left[1 + \frac{\Gamma_{\mu}}{\Gamma_{\mu}+1} \frac{B \nabla_{\mu}}{\nabla_{\rm ad} - \nabla}\right]}. \tag{18}$$

This expression differs from that given by Zahn (1992, see his expr. 3.39 with the same symbols) by the brackets at the bottom on the right side. Γ_{μ} expresses the reduction by the horizontal turbulence $D_{\rm h}$, with

$$\Gamma_{\mu} = \frac{v \ell}{6 \, D_{\rm h}}$$

where v and ℓ are a typical velocity and length. B is a thermodynamic coefficient (see expressions in Maeder 1998) which is 1.0 for perfect gas and about 0.8 in massive stars. The term ∇_μ in (18) can reduce $U(r)$ by up to 1–2 orders of magnitude, and this is a major change; previously, only the role of the μ gradient as a limit was considered (see Kippenhahn & Weigert 1990; Mestel 1953).

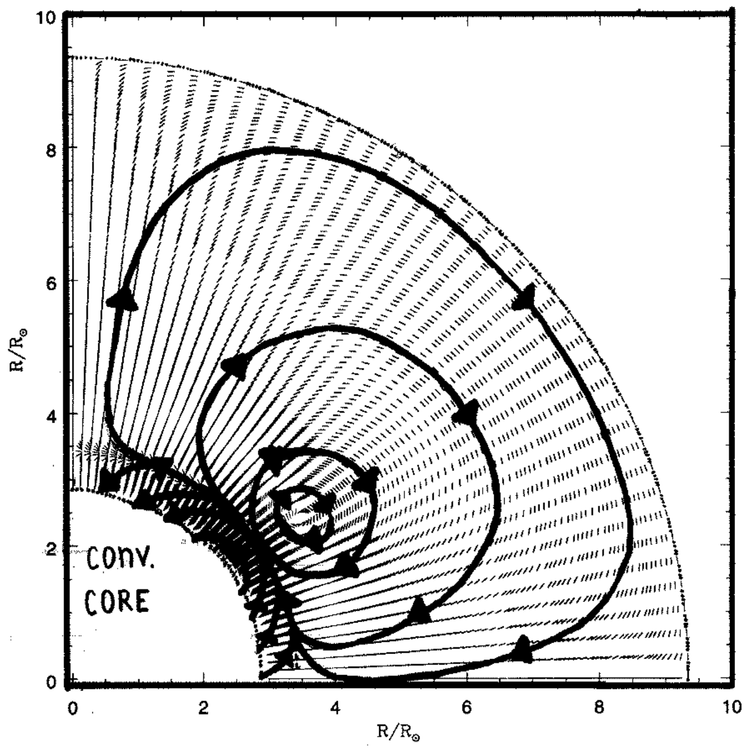

Figure 3. Illustration of the motions due to meridional circulation in a rotating star of $40M_\odot$.

Meridional circulation has an effect on the transport of both chemicals and angular momentum. As to chemicals, this transport can be treated as a diffusion. The evolution of the mass fraction X_i of an element is given by

$$\rho \frac{\partial X_i}{\partial t} = \text{nuclear terms} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho \left(D_{\text{shear}} + D_{\text{circ}}\right)\frac{\partial X_i}{\partial r}\right] \tag{19}$$

with

$$D_{\text{circ}} = \frac{\mid rU(r)\mid^2}{30\,D_{\text{h}}} = \frac{1}{30}\mid rU(r)\mid \tag{20}$$

(cf. Chaboyer & Zahn 1992). For the transport of angular momentum the basic equation is (cf. Maeder 1998)

$$\frac{\partial}{\partial t}\left(\rho r^2 \Omega\right) = \frac{1}{5\,r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega\left[U(r) - 5\dot{r}\right]\right) + \frac{1}{r^2 \partial r}\left(\rho \nu r^4 \frac{\partial \Omega}{\partial r}\right) \tag{21}$$

In the second member the term $U(r)$ is due to circulation, the term $\dot{r}$ expresses the effect of contraction or expansion, and the third term expresses the transport by shear (with ν identical to D for shear, cf. Section 4).

Equation (21) shows that the transport of Ω by circulation is not a diffusion process (if that were the case, it would involve the term $\frac{\partial \Omega}{\partial r}$). Circulation brings angular momentum from external regions, with lower Ω, to the interior, where Ω is higher (cf. Fig. 3). Thus circulation steepens the internal profile of Ω in a star, and this indirectly enhances the shear effects.

The numerical treatment of equations involving Ω deserves great care. First of all, the boundary conditions have to be written properly for the stellar surface as well as the edge of the core. A further major problem is the instability of the system of equations. The term in expression (17) contains second-order derivatives of $\Phi = \tilde{\rho}/\rho$, which itself contains a first derivative $\frac{\partial \Omega}{\partial r}$, which in turn gives a third-order derivative. Now, a derivative of $U(r)$ appears in (21), and the whole problem thus passes to fourth order! After many investigations into the numerical problems, a stable solution was obtained by applying a Henyey-type method.

6. Conclusions

At the present time we think that, in the light of Zahn's results (1992), with the various further developments discussed in the series of papers on 'Stellar Evolution with Rotation' (Paper I, Meynet & Maeder 1997; Paper II, Maeder 1997; Paper III, Maeder 1998), we actually have the most up-to-date models of rotating stars. These models show the following characteristics:

— Shellular rotation	
— Internal hydrostatic effects	
— Surface effects:	distortion change of g and T_{eff} effects on $\dot{M}$
— Shear instabilities with:	thermal losses μ gradient horizontal turbulence thermal transport by shear
— Meridional circulation with:	general equation of state μ gradient horizontal turbulence
— Transport of the elements X_i by:	convection semiconvection shear meridional circulation
— Transport of angular momentum:	by the above effects

These various effects are included in models now constructed by Georges Meynet and presented in a companion paper (Meynet 1998).

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Discussion

Langer: If the rotation is transporting angular momentum inwards, doesn't that mean that the amount of differential rotation is increasing, and that therefore the shear instability becomes larger, opposing this effect?
Maeder: Exactly! You have meridional circulation *increasing* differential rotation, but at the same time time you have more shear, and that moves angular momentum outside. So we could envisage an equilibrium between these two effects, called by Zahn the 'asymptotic solution'. As a matter of fact, there is *no* equilibrium, so this asymptotic solution cannot be applied. Moreover, at the same time μ is increasing in the centre and contraction occurs, and so, by local angular-momentum conservation, the centre is rotating faster.
Kudritzki: What is the role of mass loss in this business? It should take away angular momentum.
Maeder: Yes, this is an important point. Certainly in low-mass stars, like the Sun, there is some magnetic connection between the star and the mass lost. Thus a certain amount of mass is removing much more angular momentum than it started with. In massive stars, we don't really know much about magnetic fields, but if there is *no* magnetic field – and the usual treatments assume that – then the matter moving away carries only its own angular momentum, without depleting the angular momentum of the interior.
Townsend: Do you think that vibrational instability due to an ϵ mechanism in the semiconvective zone could have any effect on your results for the shear instability?
Maeder: These problems, such as the transport of angular momentum by waves, are being considered by Schatzmann. In principle, for massive stars we should have some vibrational instability, but current results suggest that the timescale for this instability to grow large enough to be significant is the same as the timescale for stellar evolution, so it should be unimportant.
Owocki: How important is the meridional circulation for energy transport, and can that modify the standard von Zeipel picture for gravity darkening?
Maeder: I think the von Zeipel theorem is something like the beautiful statue of King George[1]; we have to have a careful look at it [laughter]. The fact that there is a thermal imbalance with meridional circulation could certainly lead to a revision of von Zeipel's theorem.
Owocki: That hasn't been worked out yet?
Maeder: I think at least 20 years ago Lucy looked at how the von Zeipel theorem is modified if there is convection in the outer layers, but not so many people have worked on that. It's certainly a point to re-examine.

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[1]A larger-than-life equestrian statue of George III, the reigning monarch at the time of the declaration of independence of the American colonies from Britain, stands a short walk from Cumberland Lodge.