Stellar-Wind Momentum in Galaxies and a New Parameterization of the Radiative Line Force

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Abstract. We demonstrate that the classical concept of 'line-force multipliers', $\hat{k}$, α, δ, to calculate the radiative line acceleration is insufficient. General tables with these force-multiplier parameters as function of effective temperature only are bound to fail in predicting accurate wind properties across the HRD in all stages of stellar evolution. As a consequence, we develop a new parameterization method which is more precise and which will allow a much better description of radiation driven winds.

1. Introduction

One of the triumphs of the theory of radiation driven winds is the prediction of the *Wind-momentum–Luminosity Relationship* (WLR):

$$\dot{M} v_\infty R_*^{0.5} \propto L^x \tag{1}$$

and its confirmation by observations of supergiants of spectral types O, B, and A (see Fig. 1). This relationship may become the basis for a new extragalactic distance-determination method capable of reaching out to the Virgo and Fornax clusters of galaxies. It may also be used to estimate momentum and energy input into the ISM, and the strength of mass loss throughout all phases of hot-star evolution, by combining the empirical WLR with the empirical relation between terminal and escape velocities. At present, a vigorous observing programme is being carried out within a collaboration of colleagues in Caltech, Tenerife, Minnesota, and Munich, using all large telescopes available to test the WLR-method in Local-Group galaxies and to calibrate its metallicity dependence (see the corresponding contributions in this volume, and Kudritzki 1998).

Another important aspect is the quantitative theoretical interpretation of the observed WLR. Puls et al. (1996) have already shown that for O stars the theory predicts roughly the correct slope but fails to explain the difference in absolute strength between supergiants and dwarfs. A careful inspection of Fig. 1 now reveals that there may be another challenge for the theory, since there is an indication of a change in slope between O supergiants and BA supergiants. This becomes more evident in Fig. 2, where we have included the observed wind

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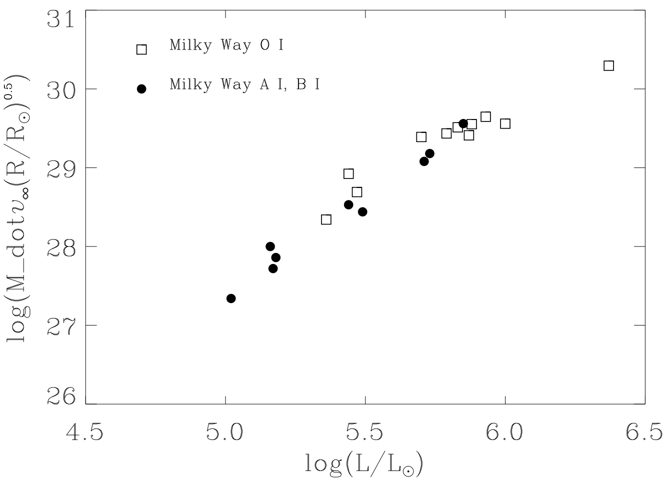

Figure 1. The observed Wind-momentum–Luminosity Relationship of A, B, and O supergiants in the Galaxy. All mass-loss rates have been determined from Hα. The terminal velocities are derived from the blue edges of UV P-Cygni profiles for the O and B supergiants and from Hα for the A supergiants. (Data from Puls et al. 1996 and Kudritzki et al. 1998; the results for the A and B supergiants are still preliminary and need final refinement.)

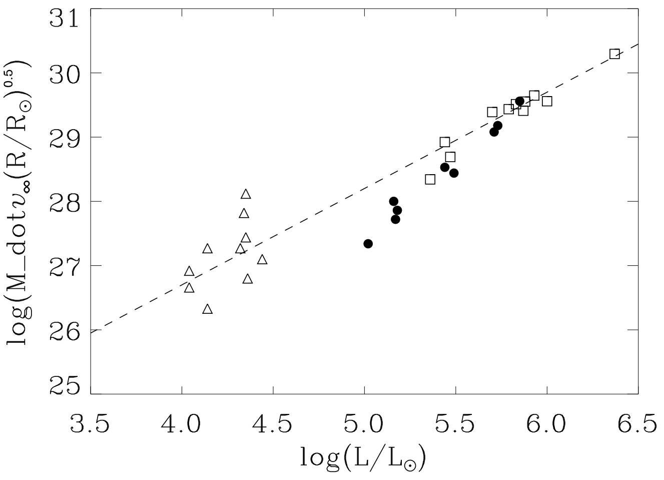

Figure 2. The observed WLR for Galactic A, B, and O supergiants compared with wind momenta of central stars of planetary nebulae (triangles), also obtained from Hα and UV P-Cygni profiles (see Kudritzki et al. 1997). The dashed line is the extrapolation of the WLR for O stars, with slope $x = 1.5$.

momenta of central stars of planetary nebulae (CSPN) as determined recently by Kudritzki et al. (1997). Strikingly, the data for the CSPN fall on the extrapolation of the WLR for the O stars. That means that for very hot stars, such as O stars or CSPN, the exponent of the WLR is $x = 1.5$, whereas for cooler objects such as A and B supergiants we observe $x = 2.5$ (see also McCarthy et al. 1997). Obviously, there is a temperature dependence in the slope of the WLR so that it becomes steeper at lower temperatures. The question is: why?

The analytical solutions for radiation-driven winds developed by Kudritzki et al. (1989) yield

$$\dot{M} v_\infty R_*^{0.5} \propto \hat{k}^{1/(\alpha-\delta)} L^{1/(\alpha-\delta)}, \tag{2}$$

where $\hat{k}, \alpha$, and δ are the (in)famous 'force-multiplier parameters' (fmps) introduced by Cast, Abbott & Klein (1975) and Abbott (1982) to calculate the radiative line force as a local quantity depending on radius, density, velocity, and velocity gradient (see also Pauldrach, Puls & Kudritzki 1986; Kudritzki, Pauldrach & Puls 1987; Pauldrach 1987; and Kudritzki 1998). These are the crucial numbers needed as function of effective temperature, luminosity, gravity, and metallicity to understand the strengths of winds in all stages of stellar evolution. The observed WLR indicates that there is a systematic variation as function of effective temperature. Lamers, Snow & Lindholm (1995) came to a similar conclusion by analyzing the observed ratios of terminal to escape velocity for O stars and BA supergiants.

To investigate the systematic behaviour of line-driven winds across the Hertzsprung-Russell diagram we have therefore started a new study of radiative line forces based on the improvements achieved during the last decade with regard to line lists and atomic physics. Here we report first results, which came as a complete surprise to us. It turns out that the classical concept of 'general' fmps $\hat{k}$, α, δ is insufficient. General tables with these fmps as function of effective temperature alone are bound to fail in predicting accurate wind properties. As a consequence, we develop a new parameterization method which is more precise and which will, in future, allow a much better description of radiation-driven winds.

The paper is organized as follows. In the next section we describe the physical assumptions leading to the classical parameterization. Then, in Section 3, we describe our new calculations of radiative line forces and show how badly they are reproduced by the classical approach. In Section 4 we introduce the new parameterization, which gives a much more precise representation of radiative line forces. In Section 5 we discuss the next steps of future work.

2. The Classical Parameterization

The crucial term in the hydrodynamics of radiation driven winds is the radiative line acceleration $g_{\rm rad}^{\rm lines}$, which can be expressed in units of $g_{\rm rad}^{\rm Th}$, the radiative acceleration provided by Thomson scattering:

$$g_{\rm rad}^{\rm lines} = g_{\rm rad}^{\rm Th}\, CF(r, v, \frac{dv}{dr})\, \mathcal{M}(t). \tag{3}$$

Here $CF(r, v, \frac{dv}{dr})$ is the finite cone angle correction factor, which takes into account that a volume element in the stellar wind is irradiated by a stellar disk of finite angular diameter rather than a point source, and $\mathcal{M}(t)$ is the *line-force multiplier,* which gives the line acceleration in units of Thomson scattering. In the Sobolev approximation the contribution of all spectral lines i at frequencies ν_i and at spectral luminosities L_{ν_i} to the line force multiplier is given by

$$\mathcal{M}(t) = \frac{v_{\rm th}}{c}\frac{1}{t}\sum_i \frac{\nu_i L_{\nu_i}}{L}(1 - e^{-\tau_i}), \tag{4}$$

where $v_{\rm th}$ is the thermal velocity of hydrogen, c the speed of light and

$$\tau_i = k_i t(r) \tag{5}$$

is the local (Sobolev) optical depth of line i computed as product of two factors, the *line strength,* k_i

$$k_i \propto \frac{n_\ell}{n_u} f_{\ell u} \lambda_i \tag{6}$$

and the *Thomson optical-depth parameter,* $t(r)$

$$t(r) = n_{\rm e}\sigma_{\rm e}\frac{v_{\rm th}}{dv/dr} \tag{7}$$

(for details, see Kudritzki 1998).

In a realistic hydrodynamic stellar-wind code the contributions of hundred thousands of lines are added up to calculate the radiative acceleration at every depth point. However, for numerical reasons these numbers are not used directly to solve the hydrodynamical problem. Instead, following the pioneering work of Castor, Abbott & Klein (1975) and Abbott (1982), $\mathcal{M}(t)$ is fitted by the parameterization

$$\mathcal{M}(t) = \hat{k}t^{-\alpha}\left\{\frac{n_{\rm e}(r)}{W(r)}/10^{11}cm^{-3}\right\}^{\delta}. \tag{8}$$

Here $\hat{k}$, α, δ are the *force-multiplier parameters* (fmps), and $W(r)$ is the geometrical dilution factor of the radiation field. This parameterization has the advantage that it allows very fast numerical solutions (see Pauldrach, Puls & Kudritzki 1986) and very precise analytical approximations of the complex hydrodynamical problem of line-driven winds (see Kudritzki et al. 1989), if one assumes that the fmps are constant in the atmosphere.

Why is it possible to approximate the sum over hundred thousands of spectral lines at different wavelengths and of entirely different strengths by such a simple parameterization? The answer is that, to some approximation, the distribution function of line strengths obeys a power law,

$$n(k,\nu)d\nu dk = (1-\alpha)g(\nu)d\nu k^{\alpha-2}dk, \tag{9}$$

at all frequencies ν (for a more detailed discussion see, for instance, Kudritzki 1998). The exponent α, which physically describes the steepness of the line-strength distribution function – or, in simpler words, the relative fraction of strong to weak lines – is mostly determined by the atomic physics of the dominant ionization stages and basically reflects the distribution function of the oscillator strengths. Typical values vary between

$$\alpha = 0.5\text{–}0.7. \tag{10}$$

If the sum in $\mathcal{M}(t)$ is replaced by a double integral (in frequency and line strength) using the line-strength distribution function, then the first two factors of eqn. (8) are obtained. The fmp $\hat{k}$ is then proportional to

$$N_{\text{eff}} = \int_0^\infty \frac{\nu L_\nu}{L} g(\nu) d\nu, \tag{11}$$

the total number of lines effectively driving the wind. Since N_{eff} changes if the ionization changes in the stellar wind, and since in non-LTE the ionization balance is, to first order, determined by the ratio of the electron density (n_{e}) to the geometrical dilution ($W(r)$) of the radiation field, the third factor is introduced. It is important to note that neglecting the ionization dependence of the force multiplier described by the third factor leads to unrealistic stellar-wind stratifications, in particular for the cooler supergiants of spectral type B and A. On the other hand, we also realize that accounting for ionization effects in the form of eqn. (8) assumes that the exponent α of the line-strength distribution function does not depend on ionization. We will see later on that this is not true.

The exciting perspective of the concept of fmps is that, if $\hat{k}$, α, δ were *a priori* known as function of effective temperature (and abundance), then mass-loss rates and terminal velocities could be immediately computed as function of temperature, luminosity, and mass in all evolutionary stages of hot stars (massive and post-AGB) using the analytical formulae of Kudritzki et al. (1989). This could be coupled with stellar-evolution codes and provide important input to understanding stellar and galactic evolution.

In the past, we have therefore tried desperately to use our hydrodynamic non-LTE radiation-driven wind code (see Pauldrach et al. 1990, 1994) to construct such a table of 'general' fmps as function of effective temperature and abundance. Unfortunately, we have failed badly. We learned that the approximation of fmps being constant within the atmosphere is too inaccurate. They are definitely depth dependent, although it is always possible *a posteriori,* after the non-LTE hydro calculation, to define a triplet of 'representative mean values' that would reproduce the calculated mass-loss rates and terminal velocities. However, these representative mean values could not be predicted *a priori* as function of stellar parameters before the detailed non-LTE hydro calculation. Thus, in the end, we were doubtful as to whether the whole concept of 'general' fmps is useful at all.

3. A Reinvestigation

Motivated by the exciting new results of Section 1 concerning the WLR, we have very recently started a reinvestigation of the fmp problem. We have used the new line list of atomic data for 2.5 million spectral lines for the non-LTE wind code used in Munich (see Pauldrach et al. 1998). In a first step, we were more interested in an investigation of the validity of the concept of 'general' fmps, rather than in a quantitatively fully correct description. We have, therefore, made several simplifying approximations to calculate $\mathcal{M}(t)$ as a sum over all spectral lines in eqn. (4). We have used analytical approximations for the calculation of non-LTE occupation numbers. For the ionization we have adopted

$$\frac{n_1^{\rm atom}}{n_1^{\rm ion}} \propto \frac{n_{\rm e}}{W} f(T_{\rm rad}, T_{\rm e}), \tag{12}$$

where $T_{\rm rad}, T_{\rm e}$ are the radiation and electron temperatures, respectively, and f is a function obtained from solving the rate equations, including ionization from and recombination to all excited levels. The excitation of subordinate levels is calculated by

$$\frac{n_u}{n_\ell} \propto W \left\{ \frac{n_u}{n_\ell}(T_{\rm rad}) \right\}_{\rm Boltzmann.} \tag{13}$$

For metastable levels we use the same formula as eqn. (13) but set the dilution factor W equal to unity. Justifications for these approximations are given by Abbott & Lucy (1985) and Springmann (1997).

For the the spectral-energy distribution we have adopted a Planck function with $T_{\rm rad} = T_{\rm eff}$. In addition, $T_{\rm e} = 0.8 T_{\rm eff}$ was assumed. It is clear that later on, after the basics of the problem with the 'general' fmps are understood, these simplifications will have to be dropped. It is also clear that this should be straightforward.

With these simplifications it is easy to calculate a very extensive grid of force multipliers $\mathcal{M}(t)$ using eqn. (4) for

- $T_{\rm eff} = 8000$–$55\,000$ K in small steps of $\Delta T_{\rm eff}$,
- in the relevant range of $\log t$ and $\log n_{\rm e}/W$ estimated from observed stellar-wind properties of massive stars and post AGB-stars.

Then we investigated whether at each effective temperature the force multipliers can be well represented by the classical fit formula

$$\log \mathcal{M}(t) = \log \hat{k} - \alpha \log t + \delta \log \hat{n}, \tag{14}$$

where

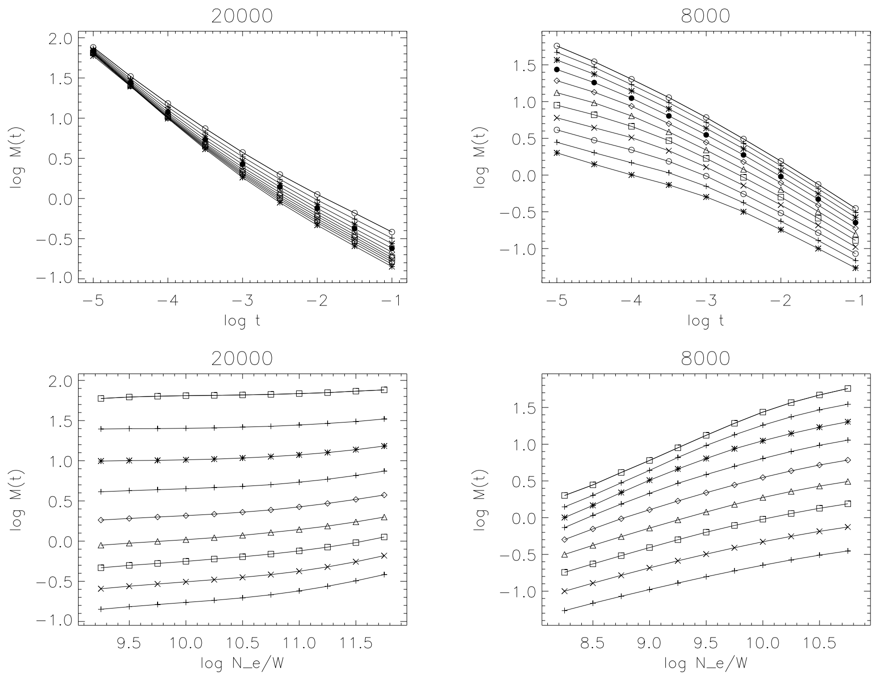

Figure 3. Force multipliers, $\log \mathcal{M}(t)$, for $T_{\rm eff} = 20\,000$ K (left) and 8000 K (right), plotted as function of $\log t$ and different constant values of $\log n_{\rm e}/W$ (upper panels) and as function of $\log n_{\rm e}/W$ for different constant values of $\log t$ (lower panels). The non-linear dependence is striking and also present at all other effective temperatures.

$$\log \hat{n} = \log \left\{ \frac{n_{\rm e}}{W} \right\} - 11. \tag{15}$$

The result was devastating. The classical fit is nowhere sufficient. There is an obvious non-linear dependence on $\log t$ and $\log n_{\rm e}/W$. This is demonstrated by Fig. 3 for two examples of $T_{\rm eff}$. This means that one either has to work with α and δ as quantities dependent on $\log t$ and $\log n_{\rm e}/W$ (see Fig. 4) or else one has to introduce higher-order fits for $\mathcal{M}(t)$.

There are two physical reasons for the non-linear dependence of $\log \mathcal{M}(t)$, as the analysis of the line-strength distribution functions shows. First, the simple power-law approximation with only one exponent is insufficient. A more detailed inspection reveals that the exponent α varies systematically as function of line strength, which introduces the curvature in the plots of $\log \mathcal{M}(t)$ as function of $\log t$. Secondly, the exponent α varies as function of ionization, which introduces curvature in the plots of $\log \mathcal{M}(t)$ as function of $\log n_{\rm e}/W$.

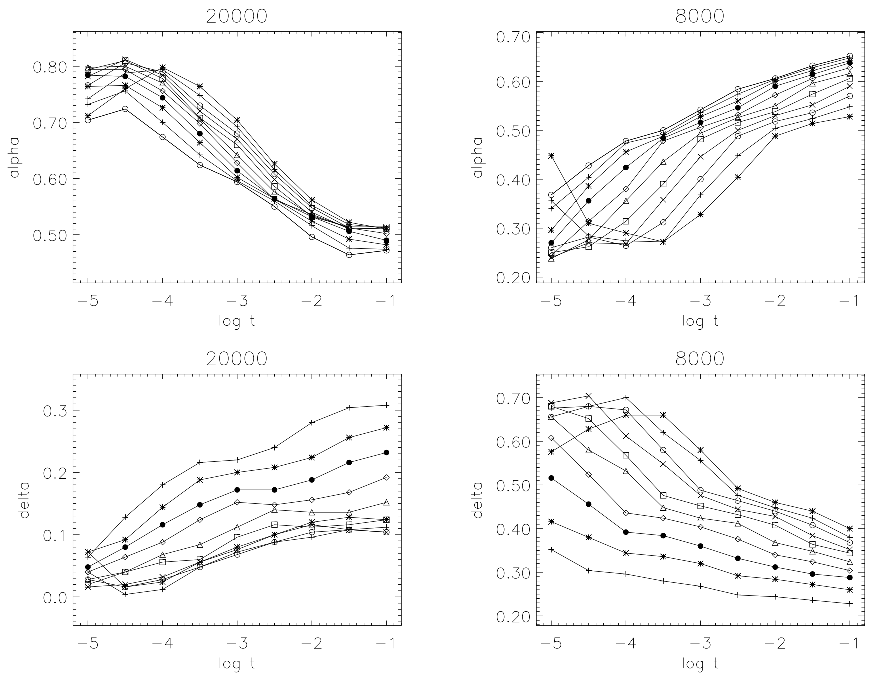

Figure 4. Fmps α (upper panels) and δ (lower panels) for $T_{\rm eff}$ = 20 000 K (left) and 8000 K (right) plotted as function of $\log t$ and different constant values of $\log n_{\rm e}/W$. The values are obtained from a pointwise differentiation of Fig. 3.

4. The New Parameterization

The simplest higher-order approach of a fit formula for the force multiplier is to assume that both α and δ depend linearily on $\log t$ and $\log n_{\rm e}/W$. With this assumption one obtains a new parameterization of the form

$$\log \mathcal{M}(t) = \log \hat{k} - \alpha_0(1+\alpha_1 \log t) \log t + \delta_0(1+\delta_1 \log \hat{n}) \log \hat{n} + \gamma \log t \log \hat{n}. \quad (16)$$

This new parameterization gives a very accurate representation of $\mathcal{M}(t)$ at every effective temperature over the full range in $\log t$ and $\log n_{\rm e}/W$. An example is given in Fig. 5. This means that we have now, in principle, found a way to describe *a priori* the radiative acceleration for every effective temperature, at all densities, before we enter a hydrodynamic calculation, which will then yield stellar-wind structures, mass-loss rates, and terminal velocities. Or – in other words – with the six force multipliers $\hat{k}, \alpha_0, \alpha_1, \delta_0, \delta_1, \gamma$ as a function of effective temperature, we can now calculate stellar-wind properties as function of luminosity, mass, etc., in all stages of stellar evolution.

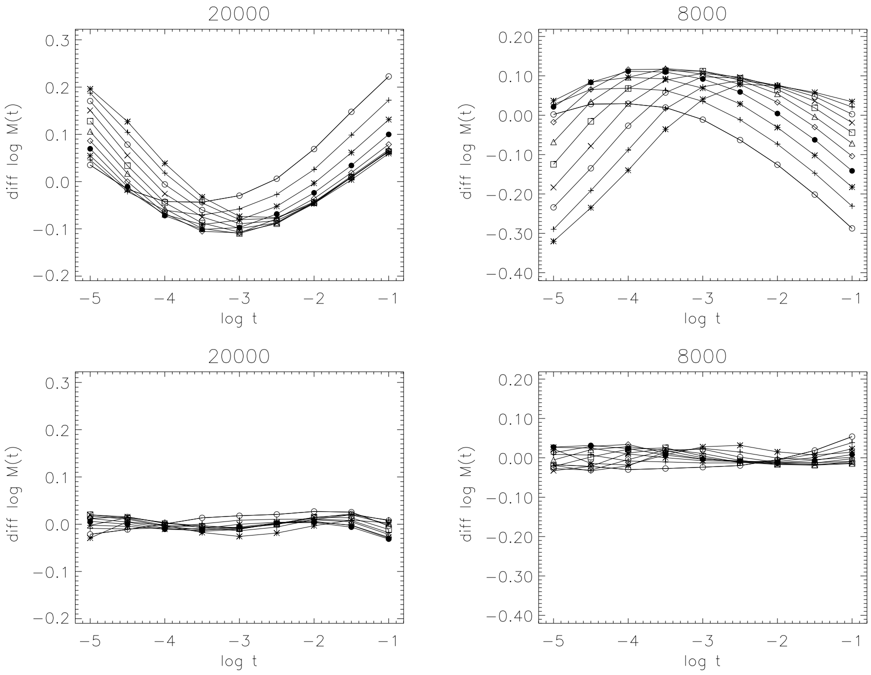

Figure 5. Logarithmic difference between the calculated force multipliers and the fit formulae $\log \mathcal{M}(t) - \log \mathcal{M}_{\rm fit}(t)$ as function of $\log t$ and different constant values of $\log n_{\rm e}/W$. Upper panels use the old force-multiplier representation of eqn. (14) for $\log \mathcal{M}_{\rm fit}(t)$, whereas lower panels use the new one of eqn. (16). Again $T_{\rm eff} = 20\,000\,{\rm K}$ (left) and 8000 K (right) have been selected as examples, but the effects are similar at all other effective temperatures.

5. Future Work

The obvious next step will be to obtain hydrodynamic stellar-wind solutions with the new representation of the radiative line force. This is not easy, but feasible. The complication arises from the fact that the analysis for finding the critical-point solutions is vastly complicated by the new form of $\mathcal{M}(t)$. However, equations have already been formulated and we expect to obtain first solutions very soon. Then we will relax several of the simplifying assumptions, such as the Planckian radiation field and the analytical non-LTE. We will carefully compare with the full non-LTE hydrodynamical stellar-wind code (see Pauldrach et al. 1994) and will finally try to develop a table for the new force-multiplier parameters as function of effective temperature and abundance. We hope that we will then be able to understand the observed WLRs and to give an improved theoretical description of stellar winds across the Hertzsprung-Russell diagram.

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Discussion

Owocki: There's the question of what happens when there are rapid jumps in line-force parameters near critical temperatures, like the bistability temperature. I didn't notice that showing up in your plots – could you comment on that? Also, is the curvature, or departure from a simple power-law dependence of the force multipliers, in the right sense to help increase the mass-loss rates in WR-star models?

Kudritzki: The critical temperatures just correspond to points at which the dominant ions of the most important elements change; that's in the input data, of course. I haven't shown plots of the parameters against temperature, but if I had then you would've seen corresponding features. The sense of curvature also depends on temperature, so there's no simple answer to the second part of your question.

Maeder: Some time ago you made calculations showing the dependence of mass-loss rates on metallicity. Do you expect very different results with your new parameterization?

Kudritzki: Not *very* different. The old calculations told you that the wind momentum goes with metallicity to some power which is a function of the force-multiplier parameters, and which is in the range of something like 0.6 to 0.8. I would have to reinvestigate this analytically with the new parameterization, but of course we can look to the detailed numerical models by Joachim Puls and Adi Pauldrach, where the force is calculated point by point. Then I think the exponent is the same – around 0.6–0.8.

The advantage of the analytical approach is first that it helps you understand the numerical models, which is not something we should forget, of course; and secondly that you can cover a very wide range of the HR diagram with a fast calculation, which you could incorporate into stellar-evolution models, for example. As an illustration, I could easily model mass loss in the post-AGB stage of evolution.

Becker: The overall slope of the power laws must depend on the number of weak lines you include, particularly at the lower end, where you seem to get the biggest differences from a linear model. Is there any problem of completeness?

Kudritzki: It's possible to calculate, analytically, the weakest lines that have to be included. In this regard, I think we are complete, although of course one can never be truly complete in the sense of including every line.

Schaerer: If I understood correctly, you seem to show that your parameterization changes completely when you go to cooler temperatures?

Kudritzki: Yes. For the O stars the old parameterization is not so bad; but still, you would not be able to explain the difference bewteen main-sequence and supergiants at the same temperature.

Schaerer: My question, really, is in regard to the B and A stars. Babel has shown that, qualitatively, the winds behave very differently in that regime, and that you have to take into account photospheric shadowing and things like that...

Kudritzki: He was looking at main-sequence objects, whose winds are very weak, and there are a number of effects at lower luminosities. Babel has shown one, and additionally Joachim Puls has found that the use of the Sobolev approximation is no longer useful for such stars [p. 245]. I'm not aiming at these objects, only at stars where the winds are really strong, like supergiants. In those the winds are strong enough to modify stellar evolution, and also I can observe them at large distances with instruments like the Keck HIRES.

Langer: Do you have a general understanding of where in the HR diagram CAK theory breaks down at low temperatures? F stars, G stars?

Kudritzki: One always has to be careful about critical-point solutions in stars with very extensive photospheres. Leaving that aside for the moment, I think that as soon as hydrogen stops being ionized, then you can't apply CAK any more. Winds in F supergiants, for example, may still be ionized, because the density is so low.