1B23
Modern Physics, Astronomy, and Cosmology

http://www.star.ucl.ac.uk/~idh/1B23

Partial Notes, Astronomy Set 1 – Stars

Luminosity and Temperature

Units

Distance: parsec 1 pc = 3.086×10^{16} m (= 3.26 light-years)

Solar luminosity: $L_\odot = 3.86 \times 10^{26}$ W

Solar radius: $R_\odot = 6.96 \times 10^{8}$ m

Solar mass: $M_\odot = 1.99 \times 10^{30}$ kg

Unit of wavelength – nanometres (or Ångstroms)
1nm= 10^{-9}m(= 10Å)

Stefan-Boltzmann Law

Luminosity:

the rate at which star emits energy.

The relation between total flux and temperature is obtained by integrating the Planck Function,

$$\pi B_\lambda = F_\lambda = \frac{2\pi h c^2 \lambda^{-5}}{\exp\{(hc)/(k\lambda T)\} - 1},$$

giving the Stefan-Boltzman law,

$$\int_\lambda F_\lambda = \sigma T^4$$

where σ is the Stefan-Boltzmann constant

(5.669×10^{-8} W m^{-2} K^{-4}).

Note 4th-power dependence on temperature

Thus the luminosity of a star is

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad \text{Watts}$$

(surface area of sphere = $4\pi R^2$)

Here the *effective temperature,* T_{eff}, is defined as the temperature of a black-body radiator that would emit the same total energy as an emitting body such as a star.

Wien's Displacement Law

The wavelength at which a BB peaks is given by $dB_\lambda/d\lambda = 0$, i.e.,

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{T} \quad \text{metres}$$

e.g., the energy distribution of Sun peaks at 500 nm, i.e., $T \simeq 5800$ K.

The radiation from a hot star with $T_{\text{eff}} = 20\,000$K peaks at 150 nm (in the ultraviolet), and at given radius is brighter at all wavelengths ($L \propto T^4$). This leads to a rationale for *multi-wavelength astronomy.*

Stellar Classification

The Harvard Spectral Classification Scheme

Spectral class	Colour	Temperature (K)	
O	Blue-violet	28,000–50,000	
B	Blue-white	10,000–28,000	
A	White	7,500–10,000	
F	Yellow-white	6,000–7,500	
G	Yellow	5,000–6,000	
K	Orange	3,500–5,000	
M	Red-orange	2,200–3,500	+ R, N (carbon stars), S
L	Brown dwarfs	1,500–2,200	
T	Brown dwarfs	$<$1,500	

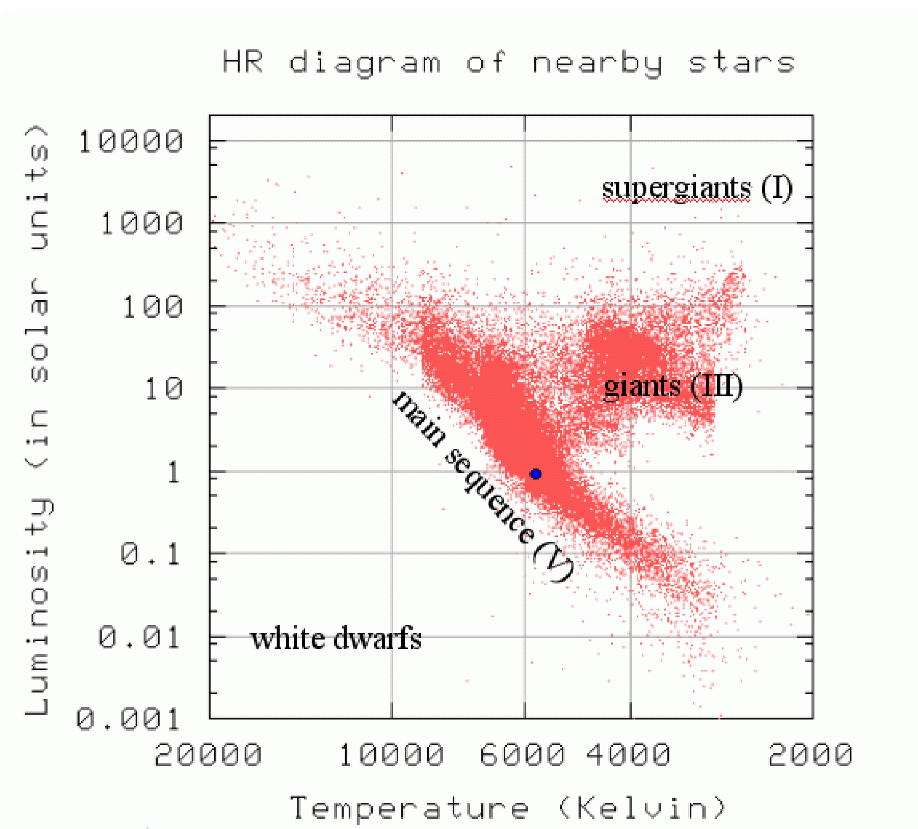
Each class is subdivided; e.g., Sun has a G2 spectral type. (Subdivisions vary pragmatically from class to class; e.g., B0, B0.2, B0.5.) The ‘earliest’ types are O2, the ‘latest’ T6 – but an evolutionary sequence is *not* implied.

Luminosity classes

Define luminosity classes I–V: I – supergiant; II – bright giant; III – giant; IV – subgiant; V – main sequence (or ‘dwarf’).

e.g., the Sun is classified as G2 V.

The Hertzsprung–Russell Diagram



Stellar Structure & Interiors

Stars are stable and very hot – entirely gaseous

A star is held together by its self-gravitation and supported against collapse by its internal pressure.

Hydrostatic equilibrium: the inward gravitational attraction exactly balances the outward pressure forces at every point within the star.

$$\frac{dP}{dr} = \frac{-GM(r)\rho(r)}{r^2}$$

where $M(r)$ and $\rho(r)$ are the mass within some radius r, and the density at that radius

(i.e., as we approach centre of star, the pressure must steadily increase to counterbalance the increasing mass above).

The central pressure and temperature of Sun
$\approx 10^{15}$ N m^{-2} and 15×10^6 K
i.e., gas at the centre is completely ionized – plasma.

Energy Production

The luminosity (energy output) of normal stars is generated by nuclear fusion. For stars on the main sequence, the most important process is the fusion of 4 hydrogen nuclei (i.e., protons) to make one helium nucleus in the central core of the star (hydrogen being by far the most abundant element – 9 out of 10 nuclei in the universe are hydrogen nuclei).

Energy Yield

Atomic weight of proton = 1.0078 amu
Atomic weight of 4 protons = 4.0312 amu
Atomic weight of He nucleus = 4.0026 amu

(where 'amu' means 'atomic mass unit' – roughly speaking, the average mass of 'nucleons' [neutrons and protons]; 1 amu= 1.66×10^{-27}kg).
The 'mass deficit' is 0.0286 amu, which is converted into energy:

$$\begin{aligned} E = mc^2 &= 0.0286 \times 1.66 \times 10^{-27}\text{kg} \times c^2 \\ &= 4.3 \times 10^{-12}\text{J} \end{aligned}$$

To compute the total thermonuclear energy available in Sun:

Assume that 10% of the mass of hydrogen is converted into helium (reactions only take place in core)

The *fractional* mass liberated in each reaction is 0.0286/4.0312 = 0.0071, so

$$\begin{aligned} E_{\text{total}} = mc^2 &= 0.0071 \times 0.1 M_\odot \times c^2 \\ &= 1.28 \times 10^{44}\text{J} \end{aligned}$$

(where the solar mass is 1 $M_\odot = 1.99 \times 10^{30}$ kg)

The lifetime of Sun = $E_{\text{total}}/L_\odot \approx 10^{10}$ yr
(where the solar luminosity $L_\odot = 3.86 \times 10^{26}$ W)

The age of solar system is 5×10^9 yr – so the Sun is roughly halfway through its so-called hydrogen burning phase.

Details of nuclear reaction

How do we build one helium from four hydrogen nuclei? The simultaneous collision of 4 protons is very, very unlikely! – so have chain reaction:

Proton-Proton Chain PP I

(1)^{1}H+^{1}H→^{2}H+e$^+$ + ν (1.18 MeV; $E_\nu \leq 0.42$ MeV)

0.25% of time, an alternative reaction occurs:

^{1}H+e$^-$+^{1}H → ^{2}H+ν ($E_\nu = 1.44$ MeV)

(2) ^{2}H+^{1}H → ^{3}He+γ (5.49 MeV)

(3) ^{3}He+^{3}He → ^{4}He+^{1}H+^{1}H (12.9 MeV)

where ^{2}H is deuterium, e$^+$ is a positron, ν is a neutrino, and γ is a photon.

Step (3) takes place 86% of time. For the remaining 14%:

(3a) ^{3}He+^{4}He → ^{7}Be+γ (1.59 MeV)

Then **PP II**

^{7}Be+e$^-$ → ^{7}Li+ν ($E_\nu = 0.86$ MeV)

^{7}Li+^{1}H → 2 ^{4}He (17.3 MeV)

or very rarely (0.02%), **PP III**

^{7}Be+^{1}H → ^{8}B+γ (0.14 MeV)

^{8}B → 2^{4}He+e$^+$ + ν ($E_\nu \leq 15$ MeV)

Detection of Neutrinos

Neutrinos produced in the PP chains escape from the Sun without interacting with any material (take a few sec).

It is therefore possible to detect them on Earth using a suitable detector and verify the thermonuclear reactions occurring in the Sun.

Homestake Mine Experiment

Started in 1965 by Raymond Davis and uses a tank containing 615 tons of tetrachloroethylene C_2Cl_4 located 1 mile underground in USA.

Solar neutrinos transmute ^{37}Cl→^{37}Ar and number of ^{37}Ar atoms ⇒ solar neutrino flux.

Detect 1 Ar atom every 2 days.

Threshold = 0.8 MeV so sensitive to PP II and PP III neutrinos.

Compare measured flux with that expected from "standard solar model" (or SSM – theoretical).

Result: measured flux is one-third to one-fourth of that expected

e.g., the chlorine experiment measures 2.55 ± 0.25 'Solar Neutrino Units (SNU)'
SSM predicts 7.9 SNU

⇒ solar neutrino problem

Kamiokande II Experiment

Two decades later, another detector called Kamiokande II in Japan has confirmed the Davis experiment.

Kamiokande II is a 3000 ton underground water detector which detects PP III neutrinos by the Cerenkov light emitted by electrons in the water when they are scattered by solar neutrinos.

Measured Kamiokande neutrino flux $= 0.45 \pm 0.15$ of SSM.

Recently, results from two more experiments using Gallium (^{71}Ga) which can detect PP I neutrinos:

$$^{71}\mathrm{Ga} + \nu \rightarrow ^{71}\mathrm{Ge} + \mathrm{e}^-$$

GALLEX (Russian) – 60 tons of metallic Ga
$\rightarrow 79 \pm 12$ SNU

SAGE (Italy) – 30 tons of Ga as Ga $\mathrm{Cl_3}$ $\rightarrow 74 \pm 14$ SNU

Both experiments give a low solar neutrino flux (SSM $\rightarrow$ 132 SNU) and confirm all previous results.

Solution: Standard Solar Model or neutrino physics?......

Energy Production, continued...

The PP chain is the dominant source of energy in low-mass stars like Sun. In more massive stars with $T_c > 18 \times 10^6$K, energy is produced by **CNO cycle**.

4H $\rightarrow$ He, using C as a catalyst.

In Sun, CNO cycle contributes 2% of solar energy.

After star has converted all H $\rightarrow$ He, further element burning.

Triple-α or Helium burning

$$^4\mathrm{He}+^4\mathrm{He}\rightarrow^8\mathrm{Be}+\gamma$$

$$^8\mathrm{Be}+^4\mathrm{He}\rightarrow^{12}\mathrm{C}+\gamma$$

Occurs for $T_c > 10^8$ K

After all He has been burnt, then have C, O, Ne, Mg burning until core is composed of ^{56}Fe (each stage requires higher temperatures and densities and thus more massive stars).

Fusion $A < 56$ releases energy
Fission $A > 56$ requires energy

Hence formation of Fe in stellar core brings fusion to an end. Hydrostatic equilibrium no longer holds – star collapses $\rightarrow$ supernova.

Stellar Evolution

Star Formation

Require a huge cloud of molecular hydrogen (H_2) that has sufficient mass to contract gravitationally.

Initially, cloud undergoes free-fall collapse as no internal pressure.

Eventually, some pressure is provided by collisions between molecules. Hydrostatic equilibrium is established and have slow collapse phase. (A 1 $M_\odot$ star takes 10^6 yr to reach this stage.)

Finally the central temperature is high enough to start hydrogen burning, the collapse stops and the star is now a 'real' star on the main sequence.

Higher mass stars arrive on main sequence with a higher luminosity and temperature.

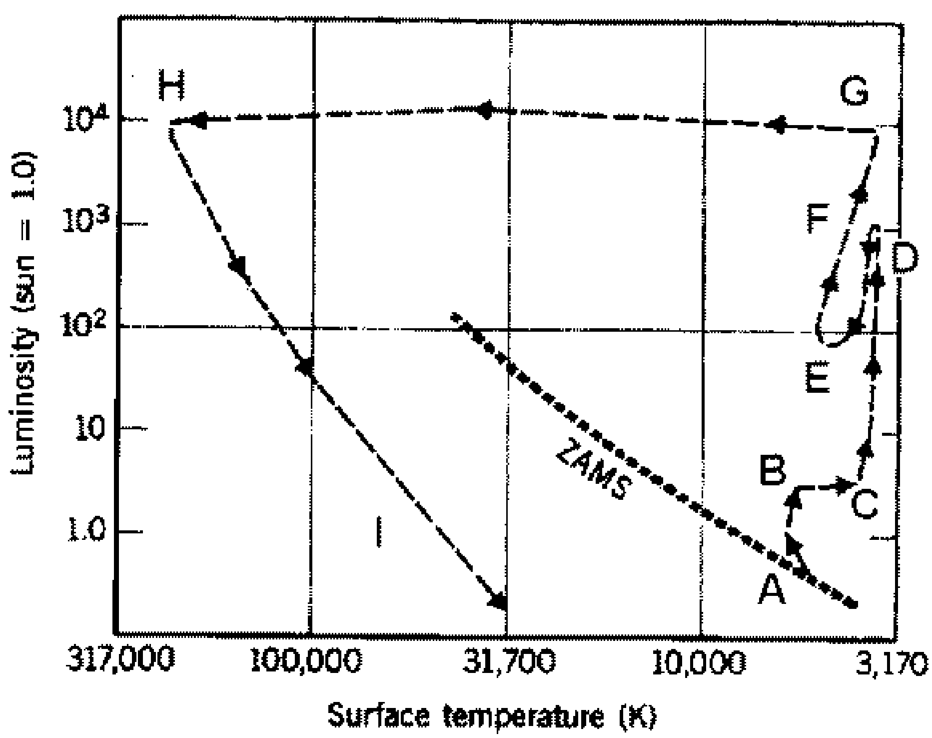
Stars with mass $< 0.08\ M_\odot$ – central temperature is never high enough to start hydrogen burning. Density in centre is so high that matter becomes degenerate which prevents further contraction. The star gradually cools off and becomes a
brown dwarf.

Main Sequence Evolution

Stars on main sequence convert H to He via PP chain or CNO cycle.
A G2V star like Sun will be initially composed of 73% hydrogen, 25% helium and 2% other elements.

The duration of main sequence ($\tau_{\rm ms}$) phase depends on the amount of hydrogen (i.e., the mass of the star) and rate at which energy is consumed (the luminosity). A more massive star will have a higher central temperature, and so the nuclear reactions will occur faster.

$\tau_{\rm ms} \sim 10^{10}$ yr for a $1M_\odot$ star and 10^7 yr for a 15 $M_\odot$ star.



Post Main Sequence Evolution of 1 $M_\odot$ Star

A star changes its composition and size as it evolves; the luminosity and surface temperature change accordingly.

On the main sequence a $1M_\odot$ star has spectral type G2 V, and converts H to He by the pp chain.

A–B:
As H in the core is converted to He, the core gradually shrinks and the internal temperature rises, causing an increase in luminosity.

B–C:
Hydrogen in the core is exhausted; the inert He core continues to contract, H-burning starts in a shell around the core, and the outer envelope expands. The surface temperature cools and star moves to right in H-R diagram.

C–D:
The luminosity increases and the star becomes a Red Giant, with radius $\sim 100\times$ greater than on main sequence.
Most of the mass is concentrated in the core, which is so dense that the electrons in the core become degenerate (Pauli's Exclusion Principle), producing a degenerate electron pressure (dependent on density but not on temperature).

D:
The core is heated by H-shell burning until it reaches $\sim 10^8$ K, when He-burning can start in a 'flash'. When part of the degenerate core is ignited, the increased temperature does *not* increase the pressure and cause the core to expand as in a normal gas. Instead, the increased temperature increases the He-burning rate, generating more energy, further increasing the temperature.

When the core temperature reaches $\sim 3.5 \times 10^8$ K, the electrons become non-degenerate and the core expands and cools. This process is called the
helium flash
because the high thermal conductivity of a degenerate gas allows He-burning to spread through the core in a few minutes.

E:
Stable He-burning phase plus H shell burning.

F:
He exhaustion – inert carbon core contracts, becomes degenerate. He and H shell burning – outer layers expand and star enters second red giant phase (the 'asymptotic giant branch', AGB).

No more core nuclear burning can occur, as the star is not massive enough to ignite carbon.

G:
He-burning (the triple-α process) is very sensitive to temperature, and He-burning shell causes star to become unstable. The star pulsates and ejects its outer layers.

H:
Planetary nebula phase – the core is exposed as a very hot white dwarf, and the ejected envelope forms a nebula which is visible for $\sim 10^4$ yr (not long!).

I:
White dwarfdom – the inert carbon core is supported by degenerate electron pressure. The star, $\approx$ the size of the Earth, gradually cools off to become a black dwarf in few $\times 10^9$ yr (n.b. 'brown dwarfs' are *not* an intermediate stage).

Evolution of More Massive Stars

On the main sequence, H-burning is by the CNO cycle. Post-main-sequence evolution is broadly similar to the 1-$M_\odot$ star, with the following important differences:

(1) Since the core temperature is much higher, burning of further elements can occur, depending on mass of star. The helium flash does not occur as the core isn't degenerate.

(2) The star goes through successive red supergiant phases (like points C–G for 1 $M_\odot$ star) between each nuclear burning phase.

(3) After each nuclear burning stage, the reaction continues in a shell around the core. At least conceptually, the star eventually has an onion-like internal structure with concentric shells, each undergoing a different reaction.

(4) For stars with initial masses > 7 $M_\odot$, nuclear burning continues until core is composed of iron, when the star collapses, generating a supernova explosion.

(5) Massive stars lose mass by a stellar wind which slows their evolution. An O star typically loses 1 $M_\odot$ in 10^6 yr.

END STATES OF STARS

The form of the stellar remnant depends on the mass of the star when the nuclear processes have finished.

Three possibilities: white dwarfs, neutron stars and black holes.

White Dwarfs

End state for stars with main sequence mass ≤ 7 $M_\odot$.

White dwarfs shine by radiating away their thermal energy until, in a few $\times 10^9$ yr, they are no longer visible at optical wavelengths and become **black dwarfs**.

White dwarfs have the unusual property that as the mass increases, the radius decreases $\rightarrow$ Chandrasekhar Limit...

Typical properties of a white dwarf:

Mass = 0.7 $M_\odot$
Radius = 0.01 $R_\odot$ = 7×10^6 m
Average density = 10^9 kg m^{-3}.

For example, Sirius B has a mass of 1.05 ± 0.03 $M_\odot$, $T_{\rm eff} = 29\,500$ K and a radius of 7×10^{-3} $R_\odot$.

Neutron Stars

For stars with main sequence masses of 7–20 $M_\odot$, and final masses of core > 1.4 $M_\odot$, the degenerate electron pressure cannot hold off gravity. Matter is crushed to such high densities that inverse beta decay occurs:

$$\mathrm{p}^+ + \mathrm{e}^- \rightarrow \mathrm{n} + \nu$$

i.e., the protons and electrons are squeezed into neutrons and a neutron gas forms which is degenerate and stops star collapsing further.

Again, neutron stars of greater mass have smaller radii so there is an upper mass limit ~ 3 $M_\odot$.

Typical properties of a neutron star:

Mass= 1.5 $M_\odot$
Radius= 10 km
Average density= 10^{17}kg m^{-3}

Escape velocity $\approx 0.8c$.

Neutron stars have very low intrinsic luminosities and can only be observed in special circumstances.

Pulsars: 'Lighthouse' model

A pulsar is a rotating, magnetic neutron star (it does **not** pulsate).

When a star collapses, angular momentum and the magnetic field strength are both conserved. Thus, a neutron star will rotate very rapidly and have a very strong magnetic field $\sim 10^8$ Tesla (Sun $\sim 5 \times 10^{-4}$ T).
As the pulsar spins, its magnetic field induces a huge electric field which rips electrons off surface.
These electrons are accelerated to relativistic speeds by rotating magnetic field lines and emit
synchrotron radiation
in a tight beam from the magnetic poles.

If the magnetic axis is not aligned with the rotation axis, but is aligned with the Earth, we will see a burst of synchrotron radiation once every rotation.

Rotational energy of pulsar goes into accelerating electrons so pulsars must gradually slow down.

Most famous example – Crab pulsar with a period of 0.03 s and a slow-down rate of $\sim 10^{-5}$ s/yr.

Black Holes

If a star is more massive than 3 $M_\odot$ (main sequence mass > 20 $M_\odot$), it will collapse beyond the neutron star stage and nothing can halt the collapse.

The volume will decrease to zero and the density will increase to infinity to form a **singularity**.

At a critical radius called the
Schwarzschild Radius
the escape velocity equals the speed of light

$$R_{\rm Sch} = 2GM/c^2$$

(for Sun $R_{\rm Sch} = 3$ km).

Also called the
event horizon
- nothing can escape.

Observing Black Holes:

The most common (only?) way to detect a stellar black hole is as member of a binary system. Any compact object in a binary system with a *lobe filling* companion will receive matter from the companion.

This material gradually spirals in towards the compact object, creating an accretion disk, which is heated and emits X-rays.

Many
X-ray binaries
are known. The masses of the two components can be measured using Kepler's third law.

In most cases, the masses are consistent with neutron stars. In a few cases the masses are much higher, suggesting a black hole.

Best known example is Cygnus X-1 which has an invisible companion with a mass of 8 $M_\odot$.

SUPERNOVAE

Massive stars are able to fuse elements up to Fe which absorbs rather than releases energy.

The iron core cannot support itself; it contracts and the temperature rises.

At $T \sim 6 \times 10^9$ K, photodisintegration of Fe occurs:

$$^{56}\mathrm{Fe} + \gamma \rightarrow 13\,^{4}\mathrm{He} + 4\mathrm{n}$$

This is an endothermic reaction which requires 100 MeV, robbing core of energy so it collapses faster.

$$^{4}\mathrm{He} \rightarrow 2\mathrm{p} + 2\mathrm{n}$$
$$\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu$$

converts core to degenerate neutron gas.

This happens in less than a second. The outer layers of the star are still collapsing and hit the hard neutron star in the centre, causing a violent rebound or shock wave to move outwards.

The outer portion of the star is blown away at speeds of 5 000–30 000 km s^{-1} to form a
supernova remnant.

Supernovae are quite rare – about 1 every 50–100 yrs in our Galaxy (although none has been observed seen 1604). The visible brightness can increase by up to a factor of 10^8, then gradually decays over several years. Seen mainly in other galaxies.

Most famous historical supernova exploded in 1054 to leave the Crab pulsar and supernova remnant.

In 1987 a star exploded in Large Magellanic Cloud and was the brightest supernova for 400 years.

SN 1987A was significant for a new branch of observational astronomy –
neutrino astronomy.

At 07:36 on 23 Feb 1987, 12 neutrinos were detected in Kamiokande II water detector over 12.4 s, and 8 neutrinos in the IMB water detector in Ohio over 5.6 s.

For the first time, the actual formation of a neutron star was observed.

Observations confirmed the standard model of stellar collapse:

Total energy in electron neutrinos $\nu_e = 10^{46}$ J.

Total visible energy over first few months $= 10^{42}$ J (i.e. much less than neutrino energy).

(Total energy generated by p-p chain in Sun over 10^{10} yr $= 10^{44}$ J.)

The neutron star has not yet been observed. It is expected to be visible in X-rays in about 30 yrs when the expanding outer layers have become optically thin and we can see through to the centre.