PHAS 3136 Problem Sheet 2 Marking Scheme.

2) From lectures

$$M_s = -M_{ISM} \log\left(\frac{p - Z_{ISM}}{p - Z_{ISMi}}\right)$$

(a) for a given $\frac{M_s}{M_{Tot}}$ the time (since start) is longer (since more ISM keeps flowing in to reduce $\frac{M_s}{M_{Tot}}$) [1]

More time $\rightarrow$ smaller fraction of stars formed early when Z_{ISM} was small [1]

(b) $$f(Z,t) = \frac{M_s(t_1)}{M_s(t)}$$ [1]

where $Z = Z_{ISM}(t_1)$

$$\Rightarrow f(Z,t) = \frac{\log\left(\frac{p - Z}{p - Z_{ISM}(t=0)}\right)}{\log\left(\frac{p - Z_{ISM}(t)}{p - Z_{ISM}(t=0)}\right)}$$ [1] either

or, assuming $Z_{ISM}(t=0) = 0$

$$f(Z,t) = \frac{\log(1 - Z/p)}{\log(1 - Z_{ISM}(t)/p)}$$

3 cont'd.

(c) $\frac{M_s}{M_{TOT}} = 0.9$; $Z = Z_\odot$

Calculate the yield from

$$M_s = -M_{ISM} \log\left(\frac{p - Z_{ISM}}{p - Z_{ISM}(t=0)}\right)$$

Assume $Z_{ISM}(t=0) = 0$

$\Rightarrow$ rearranging gives

$$1 - \frac{Z_{ISM}}{p} = e^{-M_s/M_{ISM}}$$

$Z_{ISM} = Z_\odot \Rightarrow$

$\frac{M_{ISM}}{M_{TOT}} = 1 - \frac{M_s}{M_{TOT}}$

$$p = \frac{Z_\odot}{1 \quad e^{-0.9/0.1}}$$

$$= 1.0001\ Z_\odot = 0.020002$$ [2]

Substitute to get $f(0.25 Z_\odot, t)$

$$f(Z,t) = \frac{\log(1 - Z/p)}{\log(1 - Z_{ISM}(t)/p)}$$

$$= \frac{\log(1 - 0.25 Z_\odot / 1.0001 Z_\odot)}{\log(1 - Z_\odot / 1.0001 Z_\odot)}$$

$$\doteq 0.03$$ [2]

Problem Class

13.5 Liddle.

3) a) $T = 3\times10^{28}\,k$

$\Omega_{mon} = 10^{-10}$

$\Omega_{tot} = \Omega_R = 1$

$$\rho_{mon} = \rho_{mon}(T = 3\times10^{28}k)\left[\frac{a}{a(T=3\times10^{28}k)}\right]^{-3}$$

$$\rho_R = \cancel{\rho_{R0}\,a^{-4}}\ \rho_R(T=3\times10^{28}k)\left[\frac{a}{a(T=3\times10^{28}k)}\right]^{-4}$$

find a_{eq} st. $\rho_{mon}(a_{eq}) = \rho_R(a_{eq})$

$\cancel{\rho_{mon}(T=3\times10^{28}k)}$

$\cancel{\Omega_{mon} = \rho_{mon}}$ $\quad \Omega_{mon}(a) = \dfrac{\rho_{mon}(a)}{\rho_c(a)} = 10^{-10}$ for $a = a_{GU}$

$$\Omega_R(a) = \frac{\rho_R(a)}{\rho_c(a)} \quad 1$$

$$\Rightarrow \quad \rho_{mon}(a_{eq}) = \rho_R(a_{eq})$$

$\cancel{\rho_c(a_{eq})\,\Omega_{mon}}$

$\cancel{\Omega_{mon}(a) =}$

$$\rho_{mon}(a_{GUT})\left(\frac{a}{a_{GUT}}\right)^{-3} = \rho_R(a_{GUT})\left(\frac{a}{a_{GUT}}\right)^{-4}$$

$$\frac{\rho_{mon}(a_{GUT})}{\rho_c(a_{GUT})}\ \frac{a}{a_{GUT}} = \frac{\rho_R(a_{GUT})}{\rho_c(a_{GUT})} \quad 1$$

Problem Class (2)

$$10^{-10} \frac{a}{a_{GUT}} = 1$$

$$\Rightarrow \quad a = 10^{10} a_{GUT}$$

$$T \propto \frac{1}{a}$$

$$\Rightarrow \quad T = 10^{-10} T_{GUT}$$

$$= 10^{-10}\, 3 \times 10^{28} K$$

$$= 3 \times 10^{18} K.$$

$T \simeq 3K$.

$$\frac{\Omega_{mon}(a=1)}{\Omega_R(a=1)} = \frac{\rho_{mon}(a=1)}{\rho_R(a=1)}$$

$$= \frac{\rho_{mon}(a_{GUT}) \left(\frac{a}{a_{GUT}}\right)^{-3}}{\rho_R(a_{GUT}) \left(\frac{a}{a_{GUT}}\right)^{-4}}$$

$$= \frac{\Omega_{mon}(a_{GUT})}{\Omega_R(a_{GUT})} \left(\frac{a}{a_{GUT}}\right)$$

$$= 10^{-10} \frac{a}{a_{GUT}}$$

$$= 10^{-10} \frac{T_{GUT}}{T}$$

$$= 10^{-10} \times \frac{3 \times 10^{28}}{3}$$

$$= 10^{18}$$

Problem Class (3)

Is this compatible with obs's?

$$\cancel{\frac{n_\gamma}{\Delta_B} = 10^9}$$

What is $\dfrac{\Omega_m}{\Omega_r}$ (today)?

If $z_{eq} \simeq \cancel{1000}\ 10{,}000$

$\Rightarrow$ $\dfrac{\Omega_m}{\Omega_r} \sim 10{,}000$ (check matter-rad-eq notes)

$\Rightarrow$ $\dfrac{\Omega_{mon}}{\Omega_m} \sim 10^{16}$

$\Rightarrow$ no.

UCL PHAS 3136 Problem Sheet Marking Scheme (1)

~~1~~ (a) $t_{half} \sim 60\,s$

4

$$\frac{N_n}{N_p} \sim \frac{1}{5} \exp\left(-\ln 2 \frac{t_{stop}}{t_{half}}\right)$$

$$t_{stop} = 400\,s$$

$$\rightarrow \frac{N_n}{N_p} \sim \frac{1}{5} \exp\left(-\ln 2 \frac{400}{60}\right)$$

$$\sim 0.002 \qquad [1]$$

$$Y_{He} = \frac{2 N_n}{N_n + N_p}$$

$$= \frac{2}{1 + (N_p/N_n)}$$

$$= \frac{2}{1 + 1/0.002}$$

$$= 0.004 \qquad [1]$$

cf. 0.22 for our universe.

<u>Much</u> less Helium : ~50 times less [1]

→ less easy to make planets, humans

(b) • physical Hubble length $\sim ct$ } [1]
where t = age of universe
For Hubble length at nucleosynthesis use $t \sim 1$ minute

↑ see alternative version of answer.

$\Rightarrow$ Hubble length $\sim 3 \times 10^{8}\ \mathrm{ms^{-1}} \times 60\,\mathrm{s}$

$$\sim 2 \times 10^{10}\ \mathrm{m}$$ [order 1 of magnitude]

• scale today can be found using scale factor, a, at nucleosynthesis, and

$$\text{scale today} = \frac{\text{scale then}}{a}$$

(i.e. scale today is comoving distance)

Assume matter dominated Einstein-de-Sitter i.e. $a \propto t^{2/3}$

$$\Rightarrow \quad a = \left(\frac{t_{nucl}}{t_{today}}\right)^{2/3}$$

$$\sim \left(\frac{60\,\mathrm{s}}{10 \times 10^{9} \times \underbrace{3.16 \times 10^{7}\,\mathrm{s}}_{\text{seconds per year}}}\right)^{2/3}$$

$$\sim 3 \times 10^{-11}$$ [1]

$\Rightarrow$ scale today $\sim \dfrac{2 \times 10^{10}\,\mathrm{m}}{3 \times 10^{-11}} \sim 10^{21}\,\mathrm{m} \sim \dfrac{10^{21}}{3 \times 10^{16}}\,\mathrm{pc} \sim 0.1$ [illegible]

~ galaxy to cluster size [1] [1 any units [illegible]]

UCL PHAS3136 Problem sheet 4 Marking scheme (3)

1. cont'd.

(b) <u>Alternative solution 1</u>

$$\text{Hubble length} = \frac{c}{H_0}\int_0^a \frac{a^{-2}\,da}{\sqrt{\Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}} \qquad [1]$$

$$\simeq \underset{\substack{\text{for Einstein}\\ \text{de-Sitter}\\ (\Omega_m = 1)}}{} \frac{c}{H_0}\int_0^a a^{-1/2}\,da$$

$$= \frac{2c}{H_0}a^{1/2}$$

use age of universe at nucleosynthesis to find a at nucleosynthesis

Assume matter dominated (EdS) $a \propto t^{2/3}$

$$\Rightarrow a = \left(\frac{t_{nucl}}{t_{today}}\right)^{2/3}$$

$$\sim 3\times10^{-11} \qquad [1]$$

$$\Rightarrow \text{Hubble length} \sim \frac{2\times3\times10^{8}\times(3\times10^{-11})^{1/2}\ \mathrm{ms^{-1}}}{72\times10^{3}/(3\times10^{16}\times10^{6})\ \mathrm{s}}$$

$$\sim 10^{21}\ \mathrm{m}$$

$$\sim 0.1\ \mathrm{Mpc} \qquad [1]$$

~~galaxy to cluster size [1]~~

This corresponds to 0.1 Mpc today because the eqn above gives the <u>comoving</u> Hubble length (1)

~ galaxy to cluster size (1)

3C36 Problem Sheet Marking Scheme

1. (a) Fundamental Plane eq^n given in lectures:

$$R \propto \sigma_v^{-2} L^{1.25}$$

$\sigma_v^2 \propto M/R$ from virial theorem. [1]

Substitute: $\cancel{R} \propto \frac{\cancel{R}}{M} L^{1.25}$

$$\Rightarrow \frac{M}{L} \propto L^{0.25}$$ [1]

$\Rightarrow$ More luminous galaxies (greater L) have more dark matter per star (M/L).

(b) Spirals in b-band $L \propto V_{rot}^{2.5}$ (from lectures)

Bound circular motion (gravity = centrifugal force)

gives $\frac{V_{rot}^2}{r} \propto \frac{M}{r^2} \Rightarrow V_{rot} \propto \sqrt{\frac{M}{r}}$ [1]

$$\Rightarrow L \propto \left(\frac{M}{r}\right)^{1.25}$$

Assume constant surface brightness $L = L_0 \pi r^2$

Use this to eliminate r: $r \propto \sqrt{L}$ [1]

$$\Rightarrow L \propto \left(\frac{M}{\sqrt{L}}\right)^{1.25} \propto \frac{M^{1.25}}{L^{1.25/2}}$$

$$\Rightarrow L^{(1+1.25/2)/1.25} \propto M$$

$$\Rightarrow \frac{M}{L} \propto L^{(1+1.25/2)/1.25-1} = L^{0.3}$$ [2]

$\Rightarrow$ similar to FP ellipticals