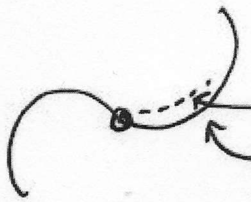
PHAS 3136 Problem sheet 2 2010 Marking scheme (7)

3 cont'd

(c) Assume width is dominated by lifetime of stars (instead of width of star formation burst) [1]



oldest stars ~ 0.1 Gyr old

youngest stars ~ 0 Gyr old

$\Rightarrow$ width = relative speed of stars (relative to spiral arm) × lifetime of stars [1]

$$= 33 \text{ km s}^{-1} \times 0.1 \text{ Gyr}$$

$$= 3.3 \text{ km} \times \underbrace{3\times10^{7} \times 10^{9}}_{\text{s Gyr}^{-1}}$$

$$\sim 10^{17} \text{ km}$$

$$\sim \frac{10^{17}}{3\times10^{16}} \text{ kpc}$$

$$\sim 3 \text{ kpc}$$ [1)

(d) consider flat rotation curve of galaxy only

$\Rightarrow$ relative speed $= \left(220 - 220 \times \frac{r}{10\text{ kpc}}\right)$ km s^{-1}

$\Rightarrow$ width $= 220 \times 0.1 \times \frac{3\times10^{7}\times10^{9}}{3\times10^{16}}\left(1 - \frac{r}{10\text{kpc}}\right)$ kpc

$$\simeq 22\left(1 - \frac{r}{10\text{kpc}}\right) \text{ kpc}$$ [2]

UCL PHAS3136 2010 Problem sheet 2 solutions

1 (a)

$f \equiv$ fraction of spirals $=$ $\dfrac{\overbrace{\text{number of spirals}}^{\equiv n_s}}{\text{no. spirals} + \underbrace{\text{no. ellipticals}}_{\equiv n_e}}$

(assume all galaxies are either spiral or elliptical)

number of galaxies with luminosity greater than L_0 $= \displaystyle\int_{L_0}^{\infty} \phi(L)\, dL$

$$n_s = \int_{L_1}^{\infty} \phi_s(L)\, dL$$

$$= \int_{L_1}^{\infty} \frac{n_{*s}}{L_{*s}} \exp\left\{-\frac{L}{L_{*s}}\right\} dL \qquad (\text{since } L_1 > L_{*s})$$

[1]

$$= \underbrace{\frac{n_{*s}}{L_{*s}}}_{\text{constants}} \left[-L_{*s} \exp\left\{\frac{-L}{L_{*s}}\right\}\right]_{L_1}^{\infty}$$

$$= \frac{n_{*s}}{\cancel{L_{*s}}} \left(\cancel{L_{*s}} \exp\left\{-\frac{L_1}{L_{*s}}\right\}\right)$$

$$= n_{*s} \exp\left\{-\frac{L_1}{L_{*s}}\right\}$$

[1]

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1 (a) cont'd (2)

$$n_e = \int_{L_1}^{\infty} \phi_e(L)\, dL$$

$$= \int_{L_1}^{L_{*e}} \frac{n_{*e}}{L_{*e}}\left(\frac{L}{L_{*e}}\right)^{\alpha_e} dL + \int_{L_{*e}}^{\infty} \frac{n_{*e}}{L_{*e}} \exp\left(-\frac{L}{L_{*e}}\right) dL$$

split integral up. ← [1]

$$= \frac{n_{*e}}{L_{*e}}\left[\frac{L_{*e}}{(\alpha_e+1)}\left(\frac{L}{L_{*e}}\right)^{\alpha_e+1}\right]_{L_1}^{L_{*e}} + \frac{n_{*e}}{L_{*e}}\left[-L_{*e}\exp\left\{-\frac{L}{L_{*e}}\right\}\right]_{L_{*e}}^{\infty}$$

$$= \frac{n_{*e}}{\cancel{L_{*e}}}\left(\frac{\cancel{L_{*e}}}{(\alpha_e+1)}\left(1-\left(\frac{L_1}{L_{*e}}\right)^{\alpha_e+1}\right)\right) + \frac{n_{*e}}{\cancel{L_{*e}}}\left(+\cancel{L_{*e}}\left(\exp(-1)\right)\right)$$

$$= \frac{n_{*e}}{(\alpha_e+1)}\left(1-\left(\frac{L_1}{L_{*e}}\right)^{\alpha_e+1}\right) + n_{*e}\exp(-1) \quad [1]$$

$$\Rightarrow f = \frac{n_s}{n_e+n_s} \quad [1]$$

$$= \frac{n_{*s}\exp\{-L_1/L_{*s}\}}{n_{*e}\left(\frac{1}{(\alpha_e+1)}\left(1-\left(\frac{L_1}{L_{*e}}\right)^{\alpha_e+1}\right)+\exp(-1)\right)+n_{*s}\exp\left\{-\frac{L_1}{L_{*s}}\right\}}$$

$$= \cancel{4}\,\frac{\exp\{-5\times10^{11}/10^{11}\}}{4\left(\frac{1}{0.5}\left(1-\left(\frac{5\times10^{11}}{10^{12}}\right)^{0.5}\right)+\exp(-1)\right)+\exp\cancel{\left(\frac{-L_1}{L_{*s}}\right)}(-5)}$$

1 (a) cont'd (3)

$f = \cancel{0.003}\ \ 0.0018$ [1]

i.e. 0.18 %.

(b) n_s is now more complicated since $L_2 < L_{*s}$

$$n_s = \int_{L_2}^{\infty} \phi_s(L)\, dL$$

$$= \int_{L_2}^{L_{*s}} \frac{n_{*s}}{L_{*s}} \left(\frac{L}{L_{*s}}\right)^{\alpha_s} dL + \int_{L_{*s}}^{\infty} \frac{n_{*s}}{L_{*s}} \left(\exp\left(-\frac{L}{L_{*s}}\right)\right) dL$$

$$= \frac{n_{*s}}{(\alpha_s+1)} \left(1 - \left(\frac{L_2}{L_{*s}}\right)^{\alpha_s+1}\right) + n_{*s} \exp(-1)$$ [1]

Same n_e as before but $L_1 \rightarrow L_2$.

$$n_e = \frac{n_{*e}}{(\alpha_e+1)} \left(1 - \left(\frac{L_2}{L_{*e}}\right)^{\alpha_e+1}\right) + n_{*e} \exp(-1)$$

$$f = \frac{\frac{-1}{0.5}\left(1 - \left(\frac{10^9}{10^{11}}\right)^{-0.5}\right) + \exp(-1)}{4\left(\frac{1}{0.5}\left(1 - \left(\frac{10^9}{10^{12}}\right)^{0.5}\right) + \exp(-1)\right) - \frac{1}{0.5}\left(1 - \left(\frac{10^9}{10^{11}}\right)^{-0.5}\right) + \exp(-1)}$$

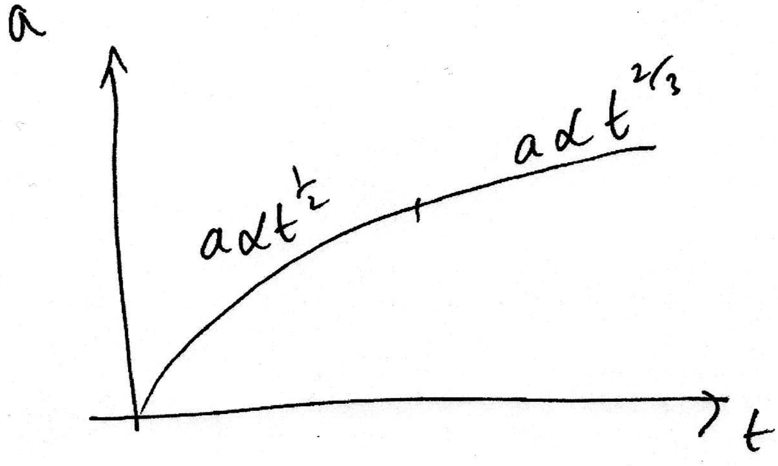
$$= \frac{18.37}{9.21 + 18.37}$$

$= 0.666 \rightarrow 67\%$. [1]

Shallow survey $\rightarrow$ dominated by ellipticals [2]

deep survey $\rightarrow$ large fraction of spirals

(e)



radiation dominated

matter dominated

[4]

2. $\rho = 0 \Rightarrow \Omega_m = 0$

$\cancel{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho + H_0^2 \Omega_k a^{-2} + H_0^2 \Omega_\Lambda}$

use $1 = \Omega_m + \Omega_k + \Omega_\Lambda$

$\Rightarrow \Omega_k = 1$ [1]

$\left(\frac{\dot a}{a}\right)^2 = H_0^2 a^{-2}$

$\Rightarrow da = H_0 dt$

$a = H_0 t + \text{const}$

$a = 0$ at $t = 0 \Rightarrow \text{const} = 0$

$a = H_0 t$ [3]

age comes from $a = 1$ today $\Rightarrow t = \text{age} = \frac{1}{H_0}$

$= \frac{1}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

$= 15$ Gyr [1]

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3

(a) $$|\Omega_{tot}(t_0) - 1| = 0.1$$

radiation dominated universe: $a \propto t^{1/2}$

$\Rightarrow \dot{a} \propto t^{-1/2}$

use result from lectures:

$$|\Omega_{tot}(t) - 1| = \frac{k}{\dot{a}^2} \propto kt$$

$$\Rightarrow |\Omega_{tot}(t_{end}) - 1| = \frac{t_{end}}{t_0} |\Omega_{tot}(t_0) - 1|$$

$$= \frac{10^{-34}s}{10 \times 10^9 \times 3 \times 10^7 s} \times 0.1$$

$$\sim 3 \times 10^{-53}$$

To order of magnitude. [3]

b) matter dominated $a \propto t^{2/3}$

$\Rightarrow \dot{a} \propto t^{-1/3}$ [1]

$$\Rightarrow |\Omega_{tot}(t_{end}) - 1| = \left(\frac{\dot{a}(t_0)}{\dot{a}(t_{end})}\right)^2 |\Omega_{tot}(t_0) - 1|$$

$$= \left(\frac{t_0}{t_{end}}\right)^{-2/3} |\Omega_{tot}(t_0) - 1|$$

$$= \left(\frac{10^{-34}}{10 \times 10^9 \times 3 \times 10^7}\right)^{-2/3} \times 0.1$$

$$\sim 10^{-35}$$ [1]

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(c) exponential expansion $\dot{a} \propto a$ [1]

$$H(t) \equiv \frac{\dot{a}}{a} = \text{constant}$$ [1]

$$|\Omega_{tot}(t_{start}) - 1| = 1$$

(d) Find $\dfrac{a(t_{start})}{a(t_{end})}$

Inflation = exponential expansion $\Rightarrow$ $\dot{a} \propto a$ [1]

$$\Rightarrow \quad \frac{a(t_{start})}{a(t_{end})} = \frac{\dot{a}(t_{start})}{\dot{a}(t_{end})}$$

$$= \left(\frac{|\Omega_{tot}(t_{end}) - 1|}{|\Omega_{tot}(t_{start}) - 1|} \right)^{\frac{1}{2}}$$

$$= \left(\frac{3 \times 10^{-53}}{1} \right)^{\frac{1}{2}}$$

$$\sim 10^{-26}$$ [2]

(i.e. universe expanded by a factor 10^{26} !)

(e) expansion time = time to expand by factor e.

$\Rightarrow$ ~100 expansion times between $t = 10^{-36}$ s and 10^{-34} s

$\Rightarrow$ expanded by factor $e^{100} \simeq 10^{43}$ [2]

much bigger than answer to (b) [1]

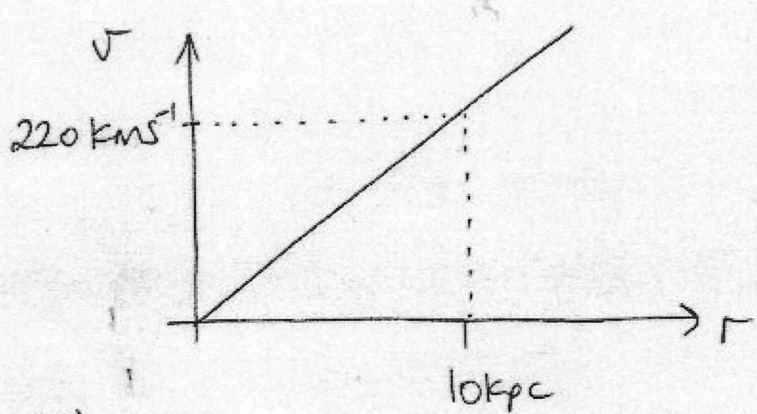
$\Rightarrow$ plenty of time to washe out initial curvature ~ 1

PHAS 3136 Problem Sheet 2 2010 Marking Scheme (6)

3(a) Spiral arms are assumed to rotate ~ as solid bodies

[Solid body rotation: $v = r\omega$
where $\omega =$ const

$\Rightarrow \quad v \propto r$



[1 mark for $v \propto r$ straight line
1 mark for $220\ \mathrm{km\,s^{-1}}$
1 mark for all remaining axis labels (v, r, 10kpc)]

(b)
~~Assume~~ Sun is at a radius of 8.5 kpc
rotation velocity of Sun (and stars) is $220\ \mathrm{km\,s^{-1}}$ } [1]

rotatation velocity of spiral arms at radius
of Sun $= 220\ \mathrm{km\,s^{-1}} \left(\frac{8.5}{10}\right)$ (since rot curve is linear)

$= 187\ \mathrm{km\,s^{-1}}$ [1]

$\Rightarrow$ stars approach spiral arms at speed

$= 220 - 187\ \mathrm{km\,s^{-1}}$

$= 33\ \mathrm{km\,s^{-1}}$ [1]