# PHAS3136 Problem Sheet 4 2010

## Due in by 3PM Monday 26 April 2010

1. (a) Draw a diagram illustrating gravitational lensing for the general case when the observer, lens and source are *not* aligned. Write down the equation relating the gravitational lensing bend angle $\hat{\alpha}$ to the impact parameter $b$. **[3]**

   (b) Draw a line joining the true and observed positions of the lensed galaxy. Hence or otherwise find an equation for the apparent change in position of the galaxy, $\alpha$, in terms of the deflection angle $\hat{\alpha}$. [Use small angle approximations throughout this question.] **[2]**

   (c) Find an equation for the angle subtended at the observer between the lens center and the apparent position of the galaxy, $\theta$, in terms of the mass of the lens $M$, the angle subtended at the observer between the lens center and the unlensed galaxy $\beta$ and the distance from the observer to the lens $D_d$, the lens to the galaxy $D_{ds}$ and from the observer to the galaxy $D_s$ (eliminating $b$, $\alpha$ and $\beta$). **[5]**

   (d) Derive an expression for the mass of the lens $M$ in terms of the image positions $\theta_1$ and $\theta_2$ and the distances $D_d$, $D_{ds}$ and $D_s$, where $\theta_1$ and $\theta_2$ are the two solutions found in part (c). **[5]**

2. (a) Imagine a new type of quasar is found which is variable on a timescale of just 1 second. Estimate the size of the emitting region. **[2]**

   (b) If this object contained a black hole, it would have a Schwarzchild radius. What limits does the fact we observe light place on the mass enclosed within the emitting region? **[4]**

   (c) Imagine the luminosity were observed to be $10^4 L_{\rm solar}$. What can we conclude from this about the mass enclosed by the emitting region (apply radiation pressure arguments)? Does this agree with your answer to the previous part of the question? **[5]**

   (d) What is the minimum radius of a dusty torus around this object? Assume that dust sublimates at 2000K. **[5]**

   [In fact this has been observed and is called a microquasar.]

3. For the following key transitions in cosmic history, perform the calculations required to fill in a table giving the approximate (i) redshift (ii) scale factor (iii) age and (iv) temperature of the Universe at the time of the transition.
(a) Matter-dark energy equality for a concordance model ($\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$). **[6]**
(b) Recombination, at a temperature of 3000K. **[2]**
(c) Matter-radiation equality, at a redshift of $\sim$3500. **[2]**
(d) Nucleosynthesis, at an energy of $\sim$1 MeV. **[4]**

[You may assume an Einstein de Sitter Universe for the age calculations, although for maximium marks calculate the age for part (d) by using the result from part (c) plus a radiation dominated Universe at earlier times.]

4. From thermodynamical arguments it is found that the energy in radiation per unit volume at temperature $T$ is given by $\epsilon_R = \alpha T^4$ where $\alpha = \pi^2 k_B^4/(15\hbar^3 c^3)$ .

(a) Using Einstein's relation between energy and mass, write down the gravitating density (effective mass) per unit volume for radiation. **[1]**

(b) For radiation at the present day temperature of $T_0 = 2.7K$ calculate the present day radiation density $\rho_{R0}$. **[2]**

(c) Evaluate the critical density today, $\rho_{c0}$ (use $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$). **[3]**

(d) Use the fluid equation to derive an equation for the density as a function of scale factor (i) for matter (ii) for radiation. [You may use $p = 0$ for matter and $p = \rho/3$ for radiation.] **[5]**

(e) Using the results found in part (d) find the redshift of matter-radiation equality in terms of $\Omega_m h^2$. **[5]**

5. If the universe were scattered with "grey dust", a hypothetical material that absorbs light equally at all wavelengths, how would this change the interpretation of the supernova observations in terms of dark energy? **[4]**

**END OF PAPER**