PHAS3136 Problem sheet 2 2010

Due in by 3pm on Friday 26 Feb 2010

1. (a) Sketch the rotation curve of the spiral arms and label the velocity at the "co-rotation radius" (the radius where the rotation velocity of the stars equals the rotation velocity of the spiral arms), which you can assume to be 10 kpc from the Galactic center. **[3]**

   (b) Estimate the speed at which stars at the radius of the Sun approach the spiral arms. **[3]**

   (c) Estimate the (tangential) width of the spiral arms, at the radius of the sun, assuming all spiral arms stars live for 0.1Gyr. **[3]**

   (d) Derive the spiral arm width as a function of radius from the center of the galaxy. **[2]**

2. The luminosity function can be approximately written as

$$\begin{aligned}\phi(L) &= \frac{n_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} \qquad \text{for} L < L_* \\ &\frac{n_*}{L_*}\exp\left(-L/L_*\right) \qquad \text{for} L > L_*\end{aligned}$$

   where for spiral galaxies $L_{*s} = 10^{11}L_\odot$, $\alpha_s = -1.5$ and for elliptical galaxies $L_{*e} = 10^{12}L_\odot$, $\alpha_e = -0.5$, where $n_{*e} = 4n_{*s}$.

   (a) A survey observes all galaxies which have a luminosity greater than $L_1 = 5 \times 10^{11}L_\odot$, within some volume. Calculate the fraction of spiral galaxies in the survey. **[6]**

   (b) Repeat this calculation for a deep survey that finds all galaxies down to a luminosity $L_2 = 10^9 L_\odot$ and comment on your result. **[4]**

3. Consider an empty universe ($\rho = 0$, $\Omega_\Lambda = 0$).

   Write down the value of the parameters Ω_m and Ω_K. **[1]**

   Solve the Friedmann equation to find how the scale factor changes with time. **[3]**

   Calculate the age of the Universe in an empty universe model. **[1]**

4. (a) If at the present day, $|\Omega_{\rm tot}(t_0) - 1| = 0.1$, calculate $|\Omega_{\rm tot}(t_{\rm end}) - 1|$ at the end of inflation ($t_{\rm end}$ 10^{-34}s). [Assume a radiation dominated universe]. **[3]**

   (b) How would your answer change on assuming a matter dominated universe? **[2]**

   (c) Find the time dependance of $H(t)$ during exponential expansion. **[2]**

   (d) If $|\Omega_{\rm tot}(t_{\rm start}) - 1| = 1$ at the start of inflation ($t_{\rm start}$), calculate the amount of inflation, $a(t_{\rm start})/a(t_{\rm end})$ required to obtain the value of $|\Omega_{\rm tot}(t_{\rm end}) - 1|$ obtained in part (a). **[3]**

(e) If inflation occurred between 10^{-36}s and 10^{-34}s and the expansion time, $H^{-1} \sim 10^{-36}$s calculate the factor by which the Universe would have expanded during this time; Compare this to the expansion factor requirement derived in (d). **[3]**

END OF PAPER