PHAS3136 Problem sheet 1 2010

Due in by 3pm on Friday 5th Feb 2010

1. Estimate the metallicity of population III stars (assume that the number of Lithium nuclei produced in the big bang is 1.6×10^{-10} of the total number; assume only H, He and Li were produced in the Big Bang; assume that a fraction 0.075 of the nuclei were helium). **[5]**

2. (a) Derive the functional form of the rotation curve expected for the outer regions of our galaxy, in the absence of dark matter (given that we see few stars at those radii). **[2]**

   (b) Derive $M(r)$ for the outer regions of our galaxy (including calculating the constants of proportionality). **[2]**

   (c) An alternative to dark matter is that the law of gravity should modified. If $F = GMm/r^{\gamma}$, what γ would be required to remove the requirement for dark matter (make the same assumptions as in the lectures). **[3]**

   (d) Derive (with constants of proportionality) the density $\rho(r)$ in the outer regions from the $M(r)$ found in part (b). **[4]**

   (e) Estimate the expected rotation curve in the center of our galaxy (assume the galaxy has constant density in the center). **[3]**

3. In this question you will derive and use the fluid equation.
   Consider an expanding sphere of material of density ρ and physical radius x.

   (a) Using Einstein's relation between energy and mass, write down the energy E in the sphere. Differentiate this to obtain an equation for the differential of the energy $\mathrm{d}E$ in terms of the differential of the volume $\mathrm{d}V$ and the differential of the density $\mathrm{d}\rho$. **[2]**

   (b) Use the first law of thermodynamics **[5]**

   $$\mathrm{d}S = \mathrm{d}E + p\mathrm{d}V \tag{1}$$

   to derive the fluid equation

   $$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) \ . \tag{2}$$

   [You may assume that the sphere is expanding slowly and thus $dS = 0$. a is the scale factor which relates comoving distance r to physical distance x through $x = ar$. Use $c = 1$ units.]

   (c) In thermodynamics courses you found that radiation pressure p_R is related to the radiation density ρ_R through $p_R = \rho_R/3$ (in $c = 1$ units). Substitute this into **[4]** the fluid equation to find how the radiation density changes with scale factor. Use the present day definitions $a = 1$ and $\rho_R = \rho_{R0}$.

(d) Write down the Friedmann equation for a flat ($\Omega_k = 0$) universe containing only radiation $\rho = \rho_R$ and no cosmological constant ($\Omega_\Lambda = 0$). Solve this to find how the scale factor depends on time (your answer will contain the Hubble constant H_0). (Use your result from part (c) and the Big Bang condition $a = 0$ at $t = 0$.) **[5]**

(e) Sketch the scale factor as a function of time for a flat universe containing just matter and radiation ($\Omega_\Lambda = 0$). **[4]**

END OF PAPER