1.3 The Galactic Rotation Curve

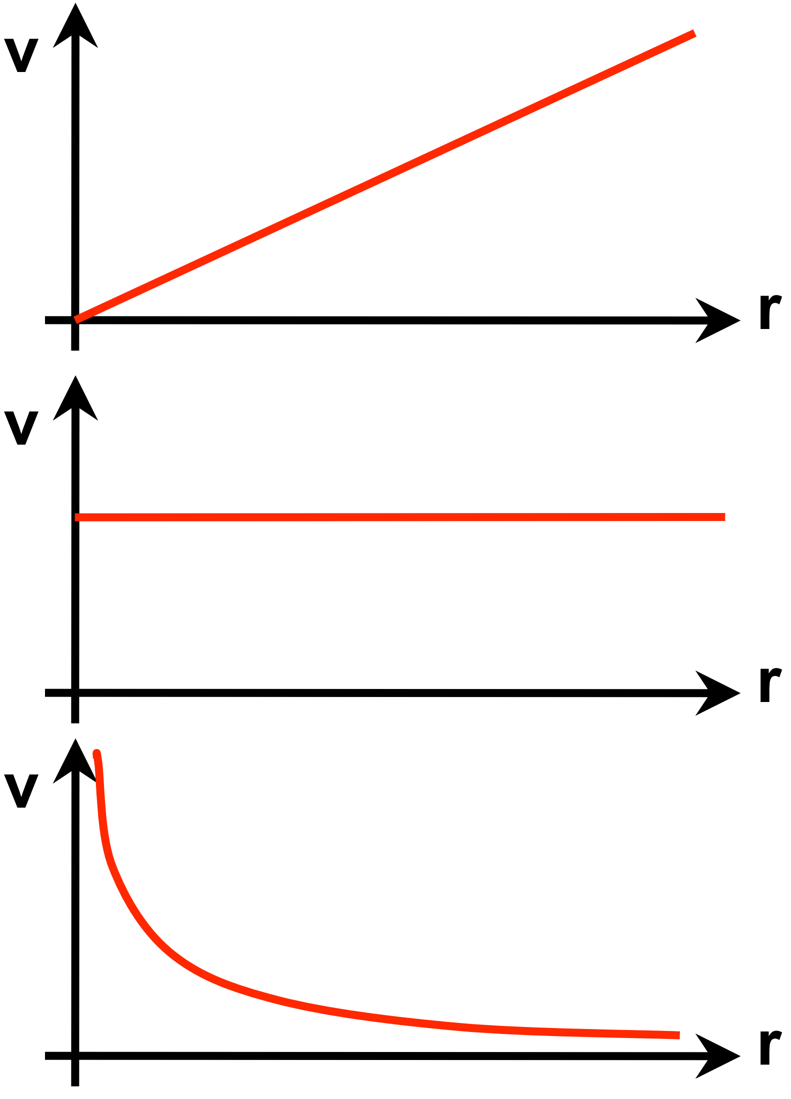
List of topics

- Keplerian rotation curve
- The rotation curve of our galaxy
- M(r) from v(r) assuming a spherical model M(r) from v(r) assuming a disk-like mass distribution
- The winding problem and density wave theory
- Metallicities of spiral arm stars
- Galactic archaeology

Some, but not all, of this material is in JL 1.3

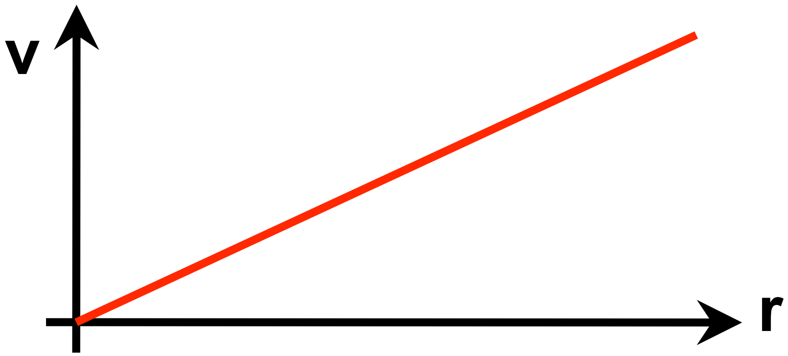
Which of the below is a Keplerian rotation curve?

1. A
2. B
3. C

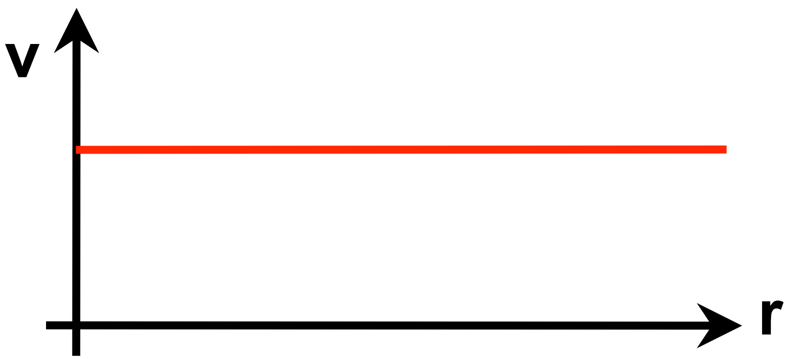


Which of the below is the rotation curve of our galaxy at large radii?

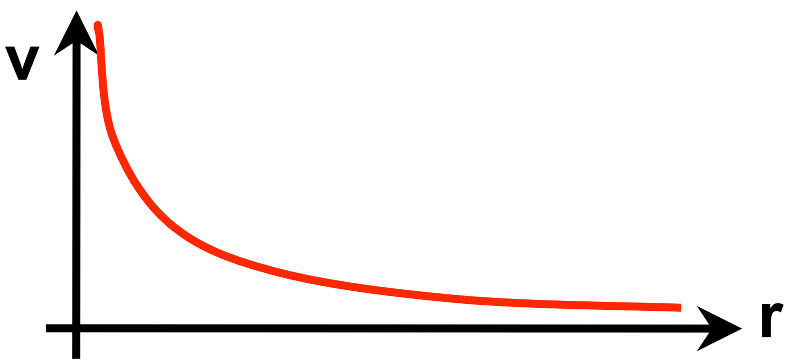
1. A



2. B

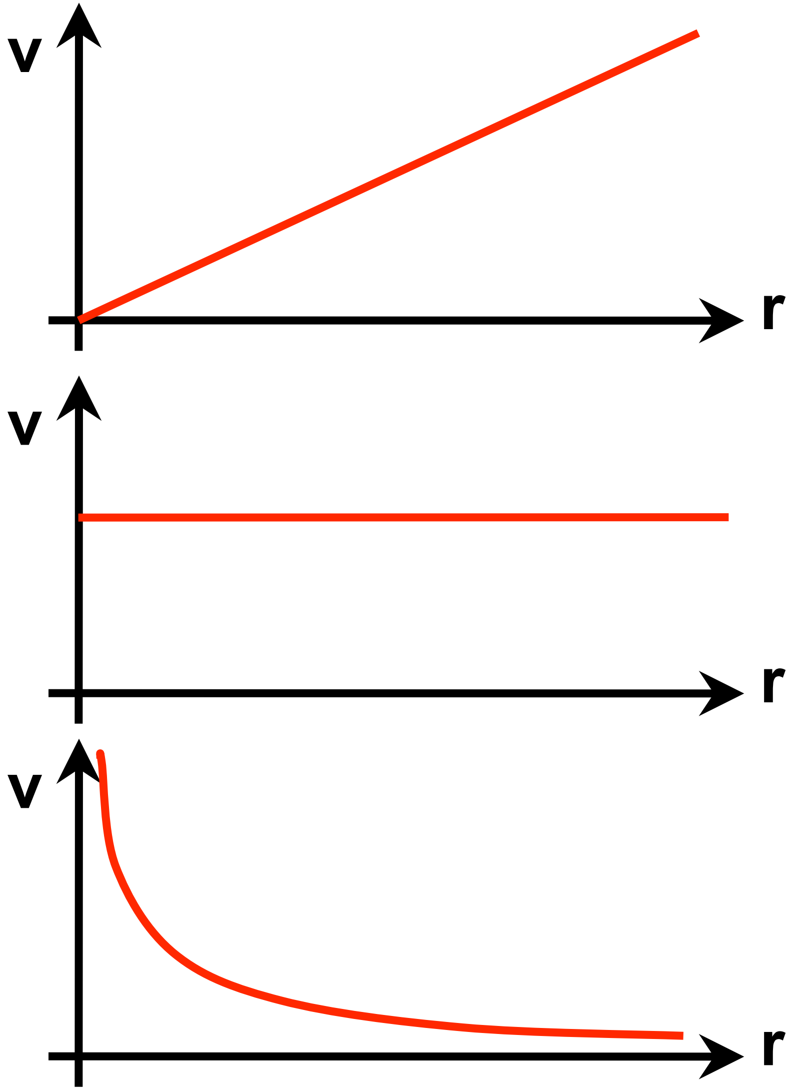


3. C



Which of the below is the rotation curve of our galaxy at *small* radii?

1. A
2. B
3. C



The rotation curve of our galaxy

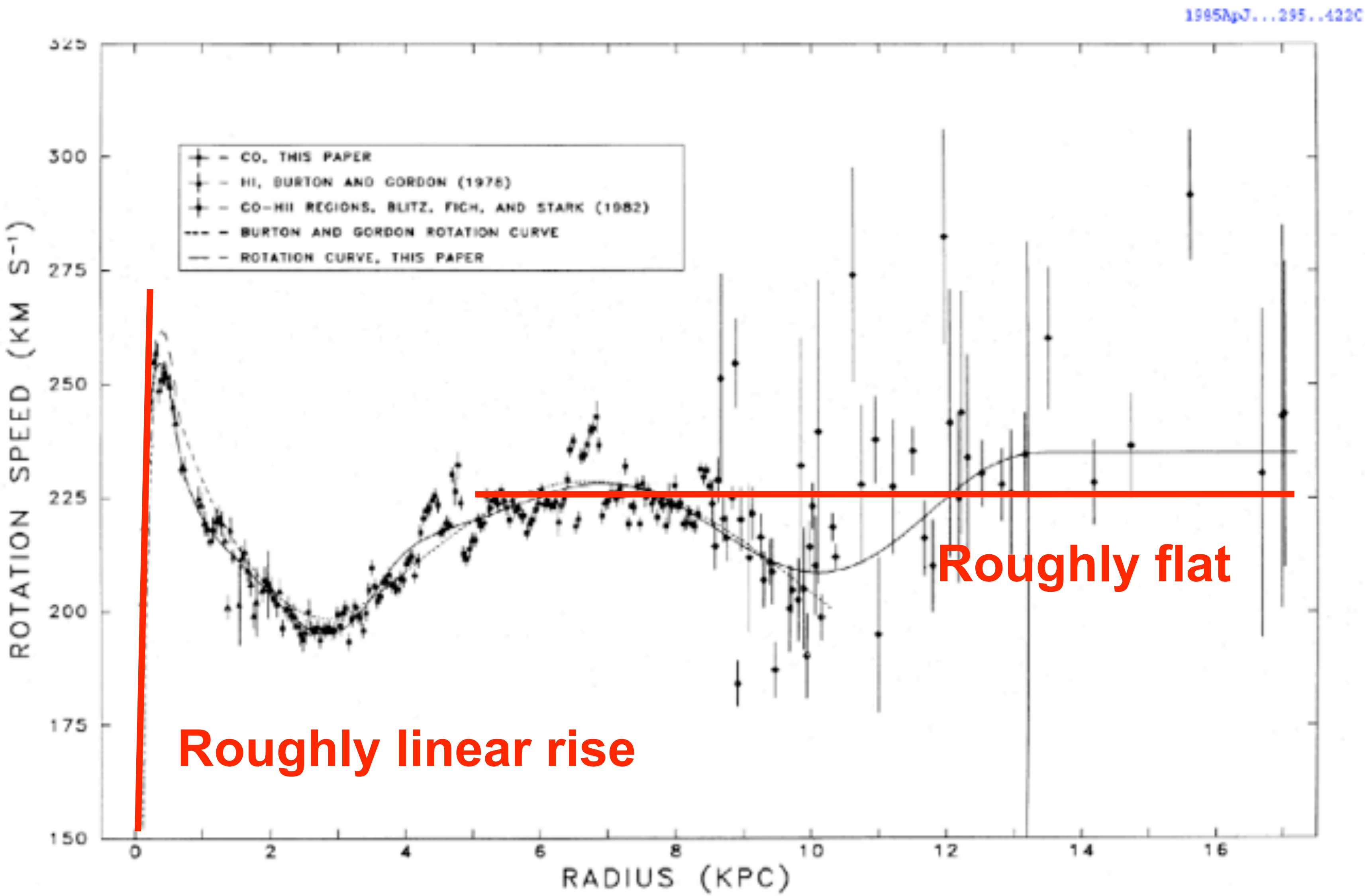
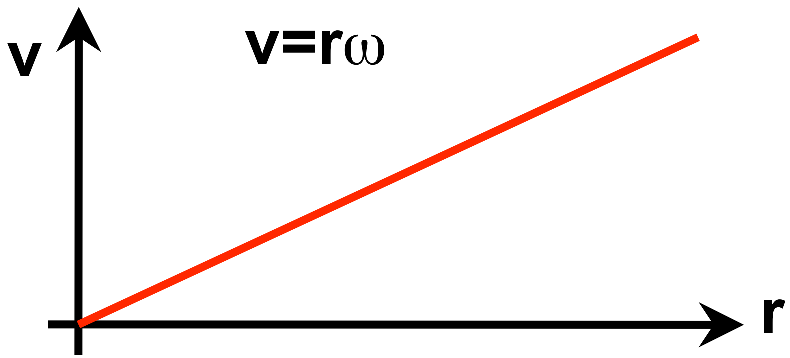


FIG. 3.—Plots of the rotation speed versus galactocentric radius. The solid lines correspond to the polynomials, and the dashed lines are the BG rotation curve. (*upper panel*)(R_0, θ_0) = (10 kpc, 220 km s^{-1}); (*lower panel*) (8.5 kpc, 220 km s^{-1}).

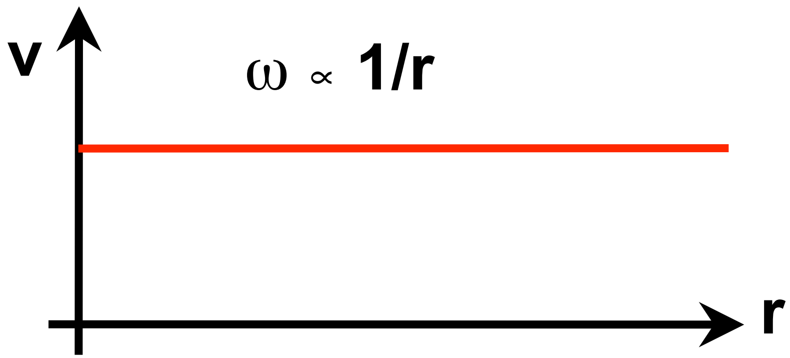
Rotation curves

- A



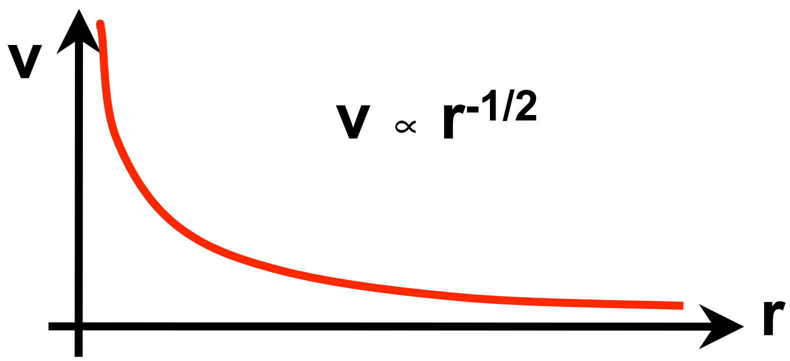
Rigid body
(~center of galaxy)
ω = constant

- B



Flat rotation curve
(Our galaxy)
Differential rotation
Not Keplerian

- C



Keplerian
(The solar system)
Differential rotation

Estimate the mass of the galaxy within r_o

Speed of light, $c = 2.998 \times 10^8$ m s^{-1}
Gravitational constant, $G = 6.670 \times 10^{-11}$ N m^2 kg^{-2}
Parsec, pc $= 3.086 \times 10^{16}$ m
Solar mass, $M_\odot = 1.989 \times 10^{30}$ kg
Solar luminosity, $L_\odot = 3.84 \times 10^{26}$ W

1. $5\times10^{9}\ M_o$
2. ★ $5\times10^{10}\ M_o$
3. $5\times10^{11}\ M_o$

You may assume the galaxy is spherically symmetric; r_o = radius of the Solar circle = 8.5 kpc

See JL Q. 1.7 (and answer)

Problem class questions

- Estimate the mass of the galaxy within r_o
  - You may assume the galaxy is spherically symmetric
  - r_o = radius of the Sun = 8.5 kpc
- Estimate M(r)
  - M(r) = the total mass enclosed in a spherical radius r.
  - You may assume $M(r) \propto r^{\alpha}$.
  - You may assume spherical symmetry.
- What does this imply for the density ?
  - If spherical $M(r) \propto r^{\alpha}$ implies $\rho \propto r^{\beta}$ where β = ?
- How could you modify gravity to remove the requirement for dark matter at large radii?
  - If $F=GMm/r^{\gamma}$ what would γ need to be?

Estimate α if $M(r) \propto r^{\alpha}$

1. -2
2. -1
3. 0
4. ★ 1
5. 2

M(r) = the total mass enclosed in a spherical radius r.
You may assume spherical symmetry.

JL Question 1.8, and answer, and page 25

Does the density increase or decrease with radius?

1. Increase
2. Decrease

What does $M(r) \propto r$ imply about the density? Suppose $\rho \propto r^{\beta}$ what is β?

1. -2
2. -1
3. 0
4. 1
5. 2

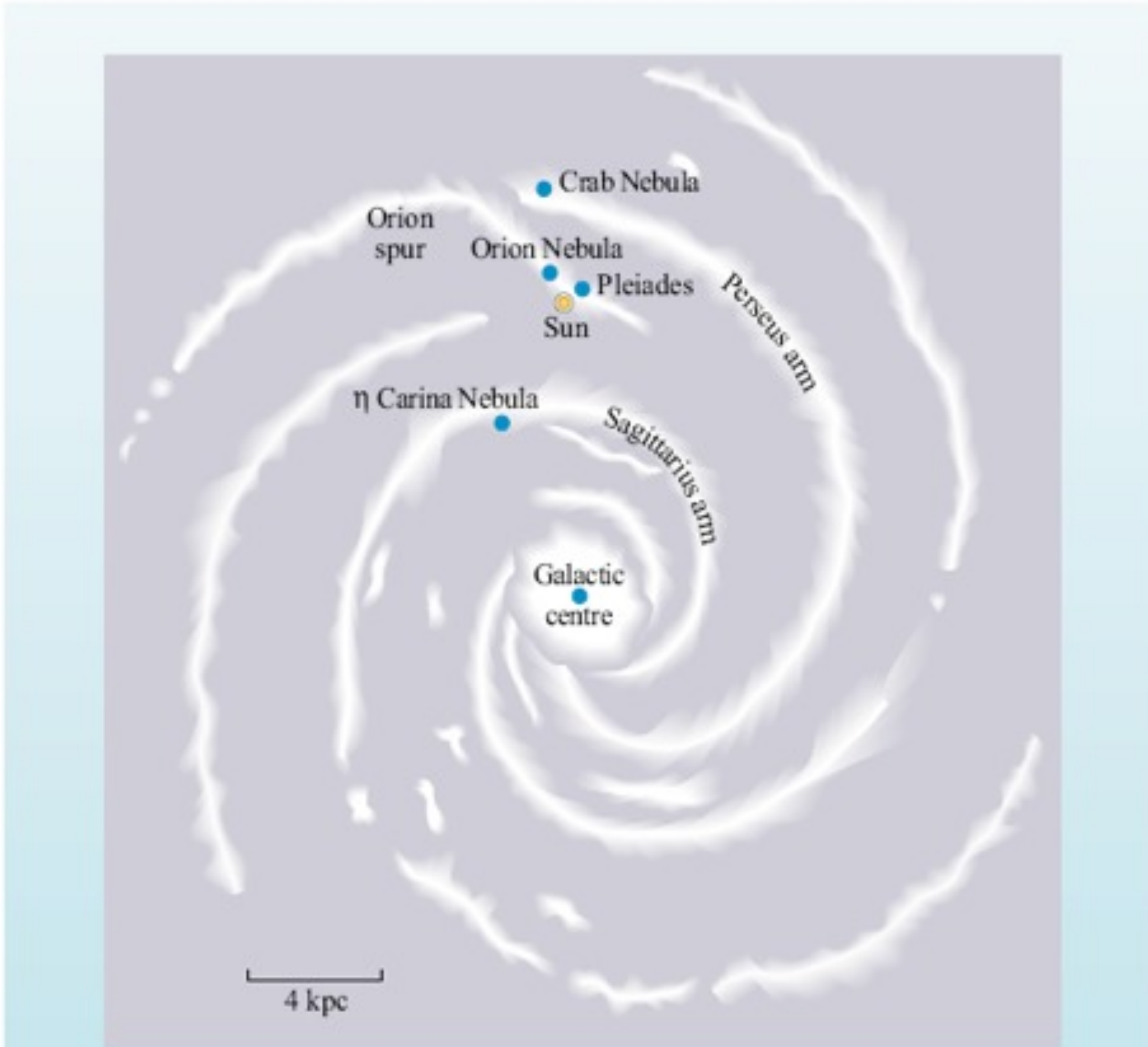


Figure 1.23 (b) Artist's schematic of the location of the Sun relative to the spiral arms. The bar/bulge is not shown in this highly schematic view.

The winding problem

- Estimate the time for one orbit of the Galactic disk at (i) 5 kpc and (ii) 10 kpc from the galactic center

Read JL p 37 - 40

Which is greater?

1. Time for 1 orbit at 5 kpc radius
2. Time for 1 orbit at 10 kpc radius

Sketch the spiral arms after ~10^{10} years

ON THE SPIRAL STRUCTURE OF DISK GALAXIES

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There are at least two possible types of spiral theories. The first alternative is to associate every spiral arm with a *given body of matter;* e.g., such an arm might essentially be a tube of gas primarily constrained by the interstellar magnetic field. The difficulty with the disrupting influence of differential rotation in such a theory is well known. The various issues associated with this point of view have been thoroughly discussed recently by Oort (1962). The second alternative is to regard the spiral structure as a *wave pattern,* which either remains stationary, or at least quasi-stationary, in a frame of reference rotating around the center of the galaxy at a proper angular speed (possibly zero).

The solution (4) is clearly of the nature of a *density wave.* Indeed, it generally has a *spiral form.* To see this, let us write (as we can always do),

There are n arms in the spiral. These are *trailing spiral arms* if $\Phi'(r) < 0$, and leading ones if $\Phi'(r) > 0$. Note that we have taken $\Omega(r)$ and n to be positive by convention. By comparing equation (7) with observed two-armed galactic spirals, it is easy to see that Φ should change by an order of 4π over a typical radial distance in such cases as the whirlpool nebula.

The contrast between the spiral patterns of Sa, Sb, and Sc galaxies can also be brought out analytically by equation (14). If there is a comparatively greater concentration of mass in the center, the density μ_0 of the disk part is relatively smaller. Equation (14) then predicts tighter spirals, as indeed are observed in Sa galaxies. More even distribution of matter corresponds to loosely wound spirals, as observed in Sc galaxies.

The total stellar population, which has various degrees of velocity dispersion, forms a *quasi-stationary spiral structure* in space of the general nature discussed above. This is primarily due to the effect of gravitational instability as limited by velocity dispersion (and secondarily to the influence of the gas and the magnetic field). The extent of density variation in the spiral pattern may be only a small fraction of the symmetrical mean density distribution.