Observational cosmology: Inflation

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Big Bang and Inflation

After these lectures, you should be able to:

- Describe the three key problems with the Big Bang model
  - The flatness problem
  - The horizon problem
  - The monopole problem
- Describe the key aspects of the theory of inflation
- Explain how inflation solves the key problems with the Big Bang
- Discuss primordial perturbations arising from inflation

Successes and failures of the Big Bang model

Successes:

- Explains expansion of the Universe
- Predicts the CMB
- Explains abundance of light elements

Failures:

- The flatness problem
- The horizon problem
- The monopole problem

The Flatness Problem

- Consider $\Omega_{tot} = \rho_{tot} / \rho_c$
  - The total physical density relative to critical at any given redshift
  - $\rho_{tot} = \rho_m + \rho_R + \rho_{DE}$

- Re-arrange the Friedmann equation:

$$\left| \Omega_{tot} - 1 \right| = \left| \frac{k}{\dot{a}^2} \right|$$

Sign is irrelevant here so take modulus

- Today: $\Omega_{tot}\,(t_0) = \Omega_m + \Omega_R + \Omega_\Lambda$
- Observations (CMB anisotropies) give
  - $|\Omega_{tot} - 1| < 0.1$
- How does $|\Omega_{tot} - 1|$ vary with time?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$

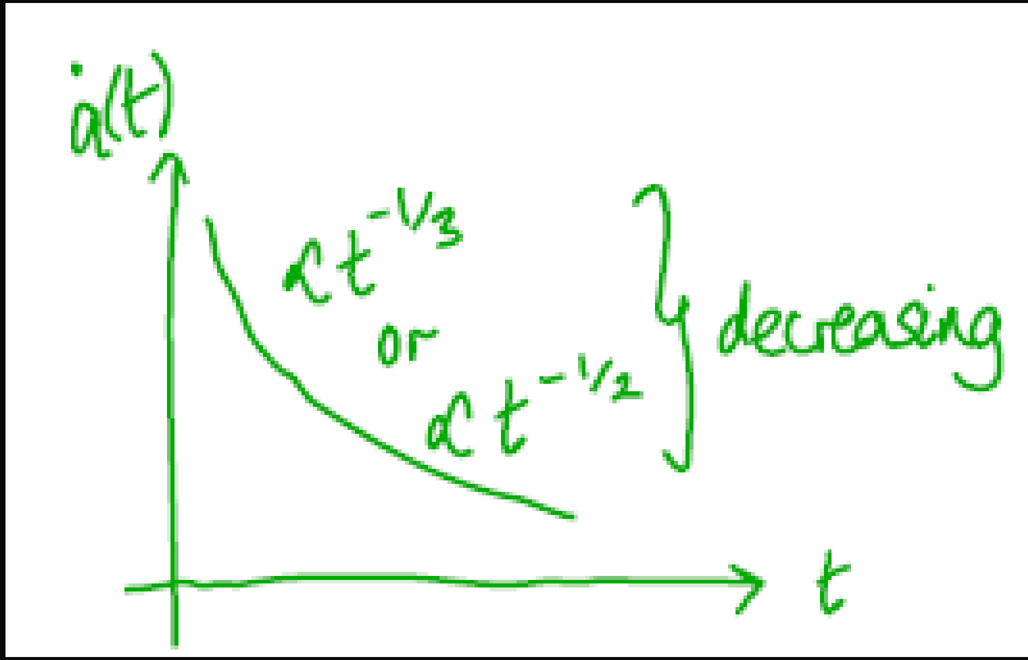
$$\Rightarrow \quad H^2 = H^2 \Omega_{tot} - \frac{k}{a^2}$$

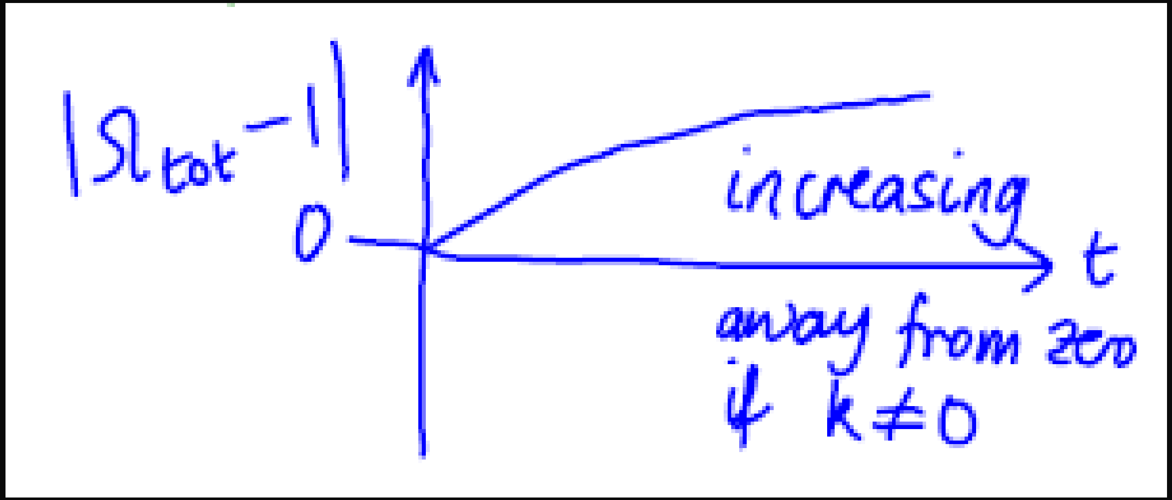
rearrange
take
modulus

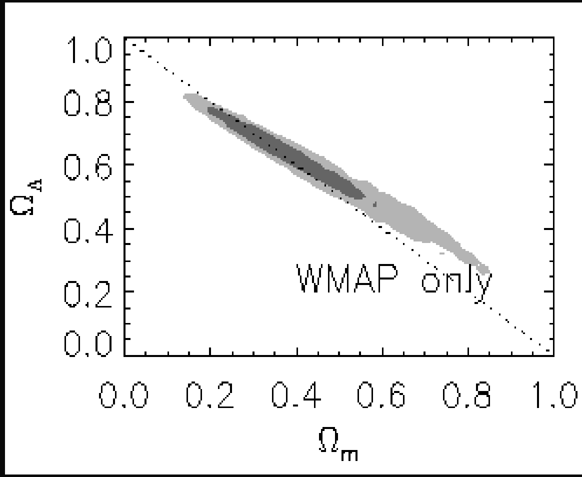
$$\Rightarrow \quad |\Omega_{tot} - 1| = \left|\frac{k}{a^2 H^2}\right|$$

$$= \left|\frac{k}{\dot{a}^2}\right|$$

- Remember how the derivative of a changes as a function of time
- So $\Omega_{tot} - 1$ goes as t to the power of 1 or 2/3 depending on matter domination or radiation domination. Either way it should increase with time
- This means that the solution omega total =1 is unstable







How bad is the flatness problem?

What was $|\Omega_{tot}(t_{nuc}) - 1|$ at nucleosynthesis ($t = t_{nuc}$) if today $|\Omega_{tot}(t_0) - 1| = 0.1$?

We have $|\Omega_{tot}(t) - 1| = \dfrac{k}{\dot{a}^2(t)}$

$$\Rightarrow |\Omega_{tot}(t) - 1| = \left(\frac{\dot{a}(t_0)}{\dot{a}(t)}\right)^2 |\Omega_{tot}(t_0) - 1|$$

eg. radiation dom: $= \left(\dfrac{t_0}{t}\right)^{-1} |\Omega_{tot}(t_0) - 1|$

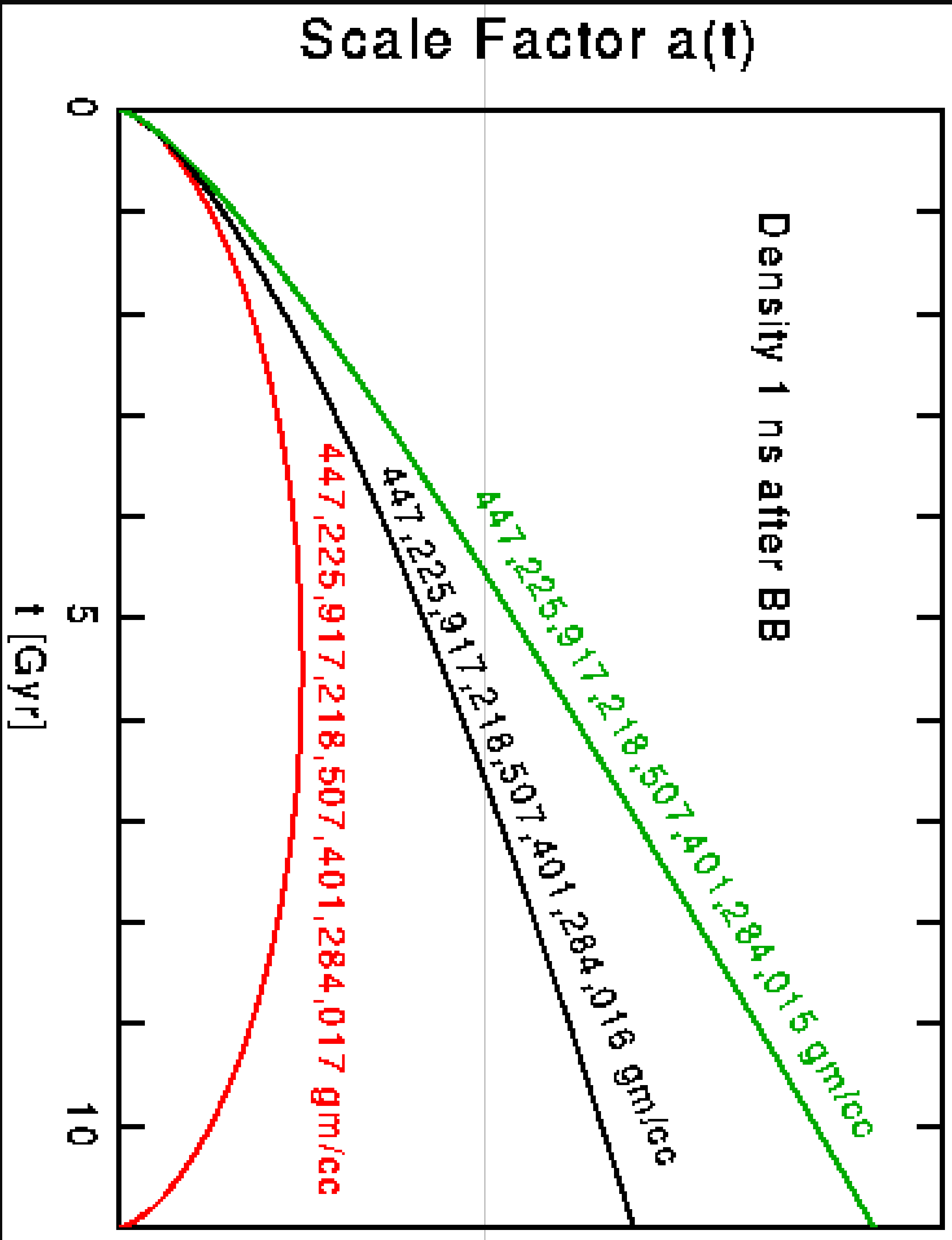
$t_0 \sim 10 \text{ Gyr} \sim 10^{17} \text{ s}$

$t_{nuc} \sim 10 \text{ s}$

$$\Rightarrow |\Omega_{tot}(t_{nuc}) - 1| \sim 10^{-16} \times 0.1$$
$$\sim 10^{-17}$$

$$0.9999\ldots\ldots9_{\times 17} < \Omega_{tot}(t_{nuc}) < 1.0000\ldots{}_{\times 17}01$$

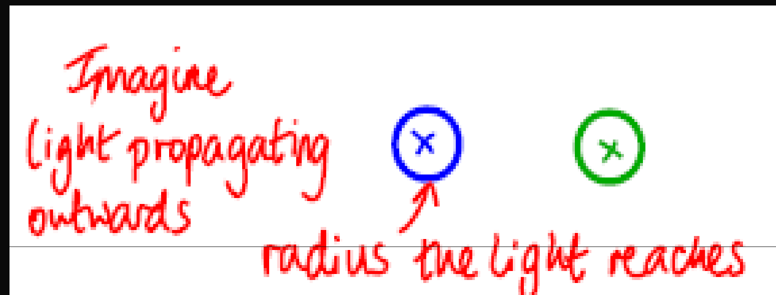
Very close to 1 !

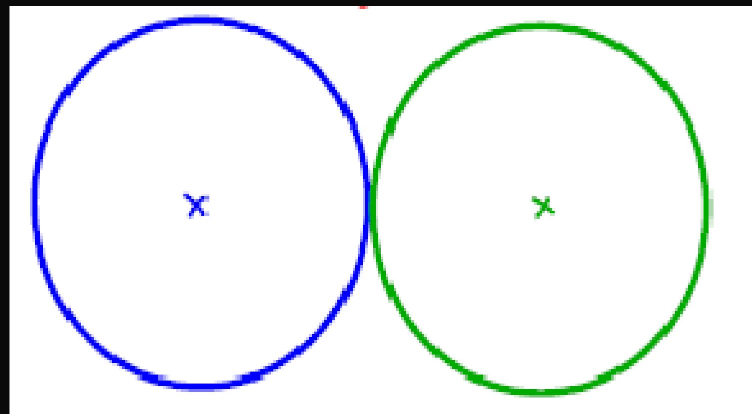


http://www.astro.ucla.edu/~wright/cosmo_03.htm

The Horizon Problem

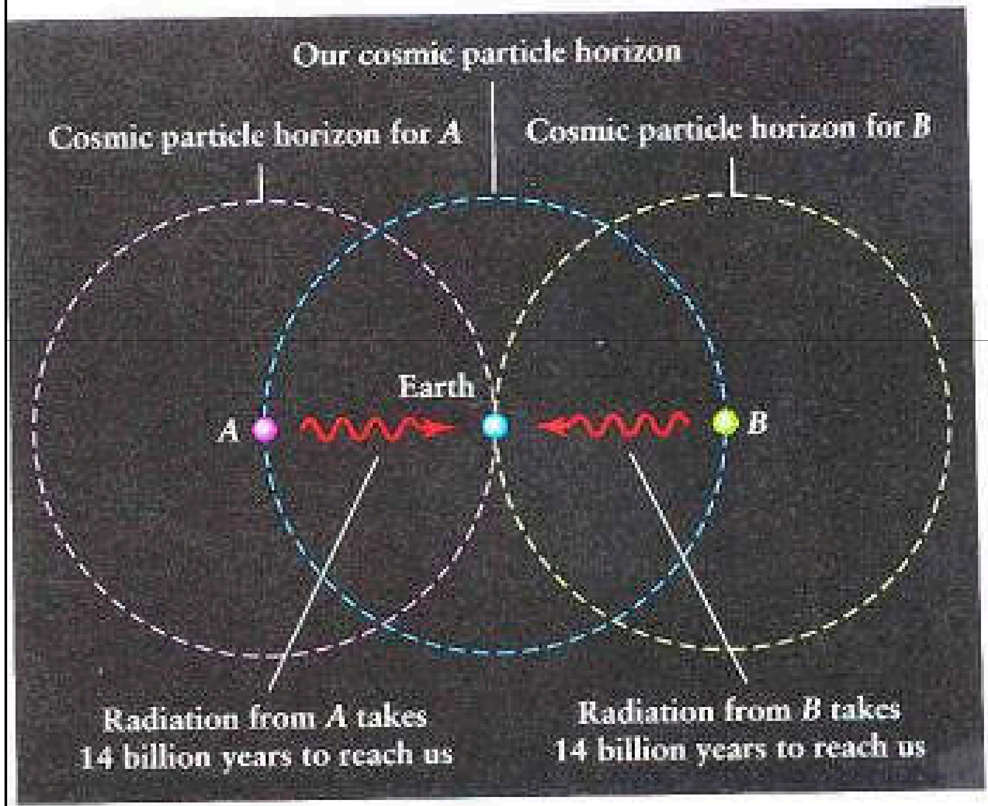
- "Horizon size": distance light could have travelled since the Big Bang
- Consider CMB photons from opposite sides of the Universe:
  - Light has not yet travelled between those points
  - Yet CMB properties are the same





- Why?
  - Would like a theory in which this arises naturally…

The horizon problem



- Light from A has no time to reach B (it takes longer than the age of the Universe)
- A and B can not communicate
- So why is the Universe isotropic?

http://physics.gmu.edu/~rms/astro113/myimages/horizon1.jpg

The Monopole Problem

- Magnetic monopole = type of particle
- Believed to be an inevitable consequence of Grand Unification Theories (GUTs)
  - Expect enormous numbers of them
  - Mass ~ 10^{16} m_H = huge
- Not observed
- Numbers predicted would easily close the Universe (not observed)
- Particle physics also predicts similar problems for other massive particles e.g. gravitinos

Inflation

- ≡ Phase of rapid acceleration
  - Proposed as an add-on to the Big Bang model (Big Bang Inflation model)

- Could be caused by something like a cosmological constant
  - Often called the "inflaton"
  - Leads to exponential expansion

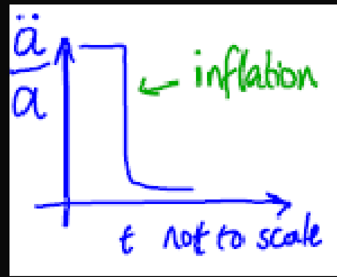
- Inflation must have ended
  - e.g. to allow gravity to form galaxies

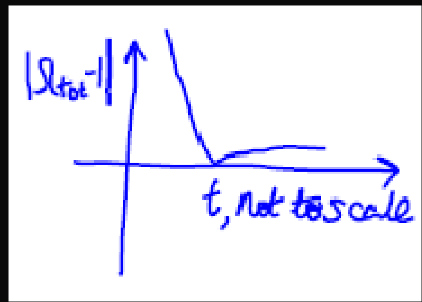
- Thought to happen very early
  - $t \sim 10^{-34}$ s

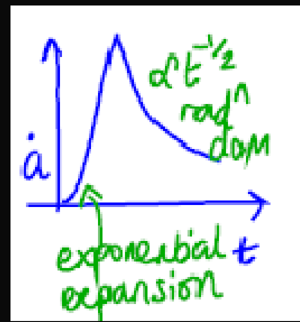
  ⇒ Negligible changes to calculations we already did

Implications of Inflation

- Flatness problem:
  - Rapid accel -> adot grows rapidly
  - $|\Omega_{tot} -1|$ reduces dramatically
  - Analogy:
    - Blow up surface of balloon
    - Looks flatter



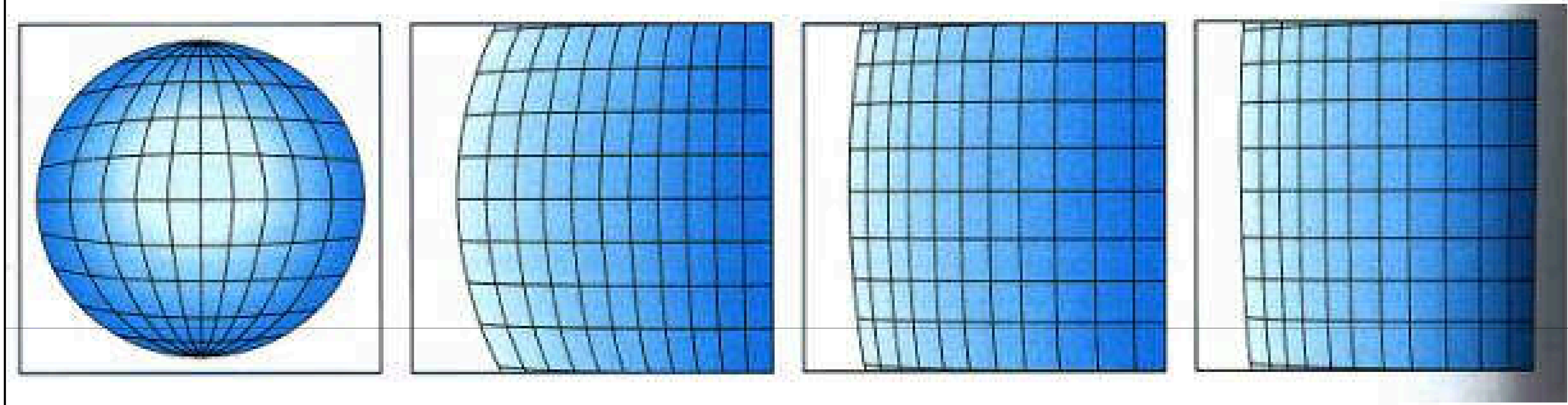




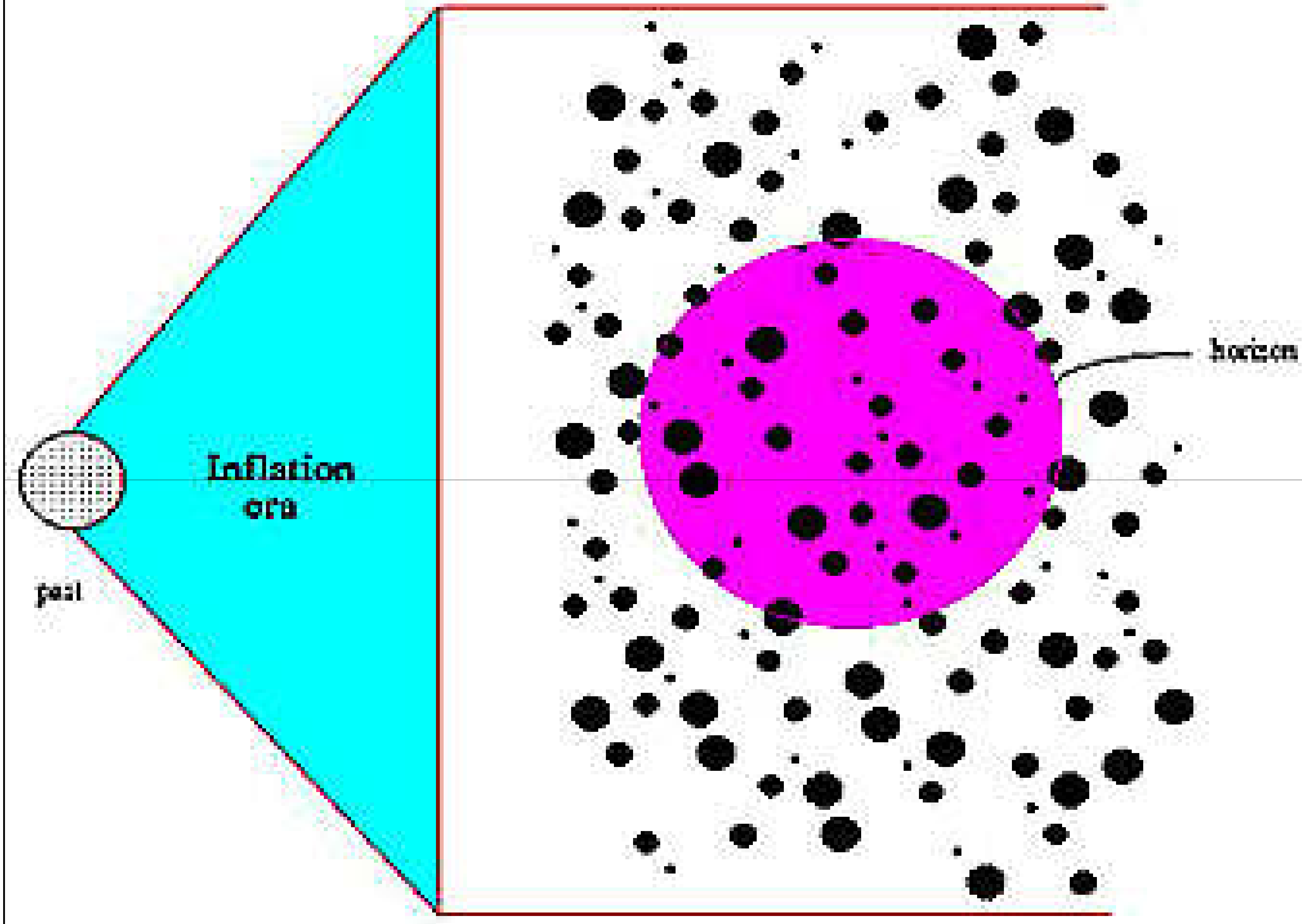
- Horizon problem:
  - Small thermalised patch of Universe is inflated beyond the current horizon
  - We see homogeneity

- Monopole problem:
  - Number density of monopoles is dramatically reduced

Inflation solves flatness



- Sphere inflated by factor 3 in each frame
- Curvature becomes undetectable on scale of figure
- Inflation can produce locally flat Universe



http://rst.gsfc.nasa.gov/Sect20/inflationX.jpg

Derive $a(t)$ for a Universe dominated by a cosmological constant (assume $\Omega_k = 0$)

- $\Omega_\Lambda \neq 0$, $\Omega_m = 0$, $(\Omega_R = 0)$, $\Omega_k = 0$
- Friedmann eqⁿ: $\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda$
- Solve for $a(t)$:

$$\frac{\dot{a}}{a} = H_0 \Omega_\Lambda^{1/2}$$

$$\frac{da}{a} = H_0 \Omega_\Lambda^{1/2} \, dt$$

$$\ln a = H_0 \Omega_\Lambda^{1/2} t + Q \quad \leftarrow \text{const}$$

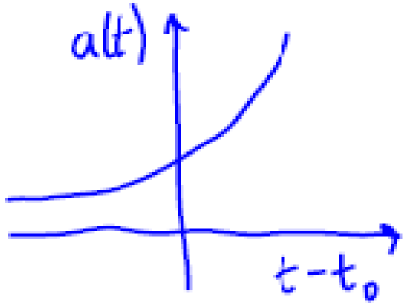
$$a = e^Q e^{H_0 \Omega_\Lambda^{1/2} t}$$

- Set constant by considering the present day:

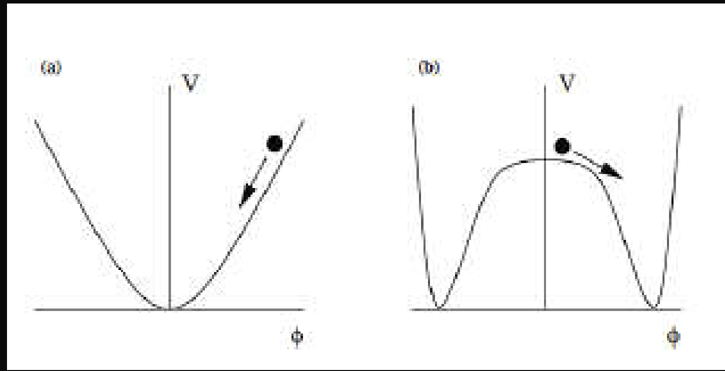
$a = 1$ at $t = t_0$ $\Rightarrow$ $1 = e^Q e^{H_0 \Omega_\Lambda^{1/2} t_0}$

$$\Rightarrow e^Q = e^{-H_0 \Omega_\Lambda^{1/2} t_0}$$

$$\Rightarrow \quad a = e^{H_0 \Omega_\Lambda^{1/2} (t - t_0)}$$



Inflation needs scalar fields:



- For a field phi that exists in space

$$T^{\mu\nu} = \partial^\mu \phi \, \partial^\nu \phi - L g^{\mu\nu}$$

$$L = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - V(\phi)$$

where L is the lagrangian.

- Comparing to a fluid as before we have:

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}(\nabla\phi)^2 \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{1}{6}(\nabla\phi)^2 \end{aligned}$$

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

- Equivalent of Friedman eq:

$$\ddot{\phi} + 3H\dot{\phi} - \nabla^2\phi + V' = 0$$

- If V(Φ) bigger than the kinetic term slow roll (see later):

Derive the Friedman equation for fields.

- We start with the fluid equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

- We replace the density and pressure for scalar fields in.

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}(\nabla\phi)^2 \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{1}{6}(\nabla\phi)^2 \end{aligned}$$

- The gives:

$$\dot{\phi}\ddot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}t} + 3H\dot{\phi}^2 = 0$$

- Dividing by the derivative of phi:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

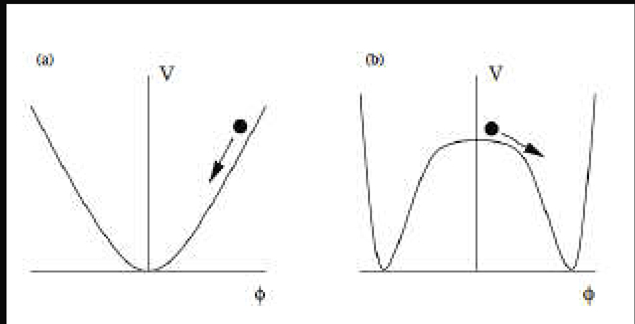
Slow roll and ending inflation:

- Slow roll parameters:

$$|\eta| \equiv \left|\frac{m_{pl}^2}{8\pi}\frac{V''}{V}\right| \ll 1$$

$$\epsilon \equiv \frac{m_{pl}}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1$$

- Inflation ends when the field is near the bottom and slow roll is no longer satisfied.



- The Universe reheats
- The filed can oscillate being damped by 3H term
- Number of e-foldings is:
  - Need about 60 e-folds…

$$N = \int H\,dt = -\frac{8\pi}{m_{pl}^2}\int_{\phi_{init}}^{\phi_{final}} d\phi \frac{V}{V'}$$

Show that this is equivalent to lambda expansion, i.e. exponential

- Starting from:
- The friedman equation is equivalent to:
- Assume slow roll. This is equivalent to the first term in the first equation to be negligible and the derivative of the filed in the second equation.
- Replacing one of the H into the first equation:
- Rearranging and removing the time dependence:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

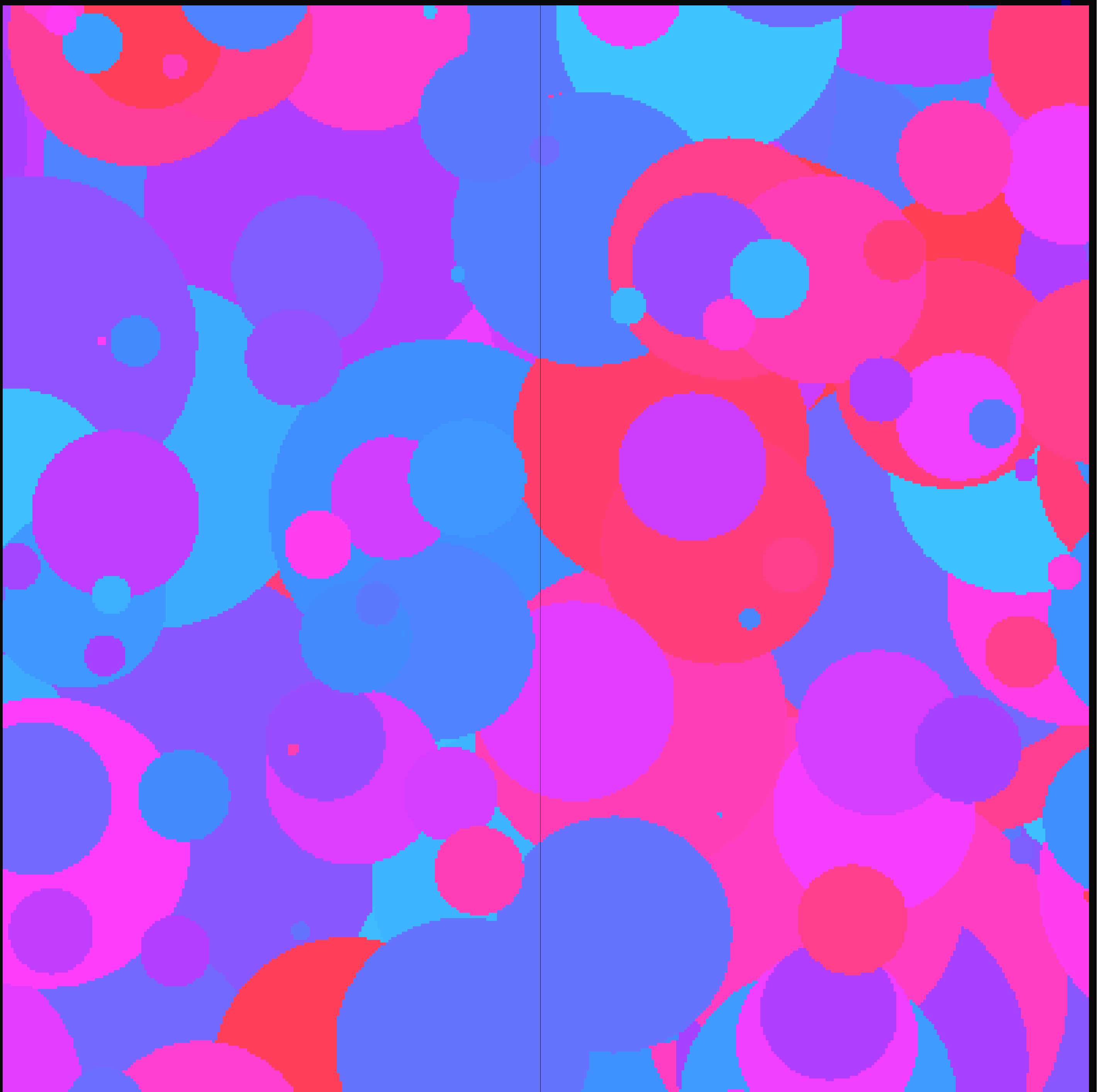
$$H^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}}{2} + V\right)$$

$$\frac{8\pi G}{H} V\dot{\phi} + V' = 0$$

$$\frac{\mathrm{d}a}{a} = -8\pi G \frac{V}{V'} \mathrm{d}\phi$$

Inflation produces density perturbations

- Pointed out 1 year after inflation was proposed.
- Tiny quantum fluctuations are blown up to large scales
- Tiny quantum fluctuations continue to be produced during inflation
  - All scales have a similar amplitude of fluctuations just after inflation
- Gravity amplifies these fluctuations into galaxies and clusters of galaxies…
- Seems to fit observations well, possibly the most convincing success of inflation.



http://www.astro.ucla.edu/~wright/CMB-MN-03/11Oct03/

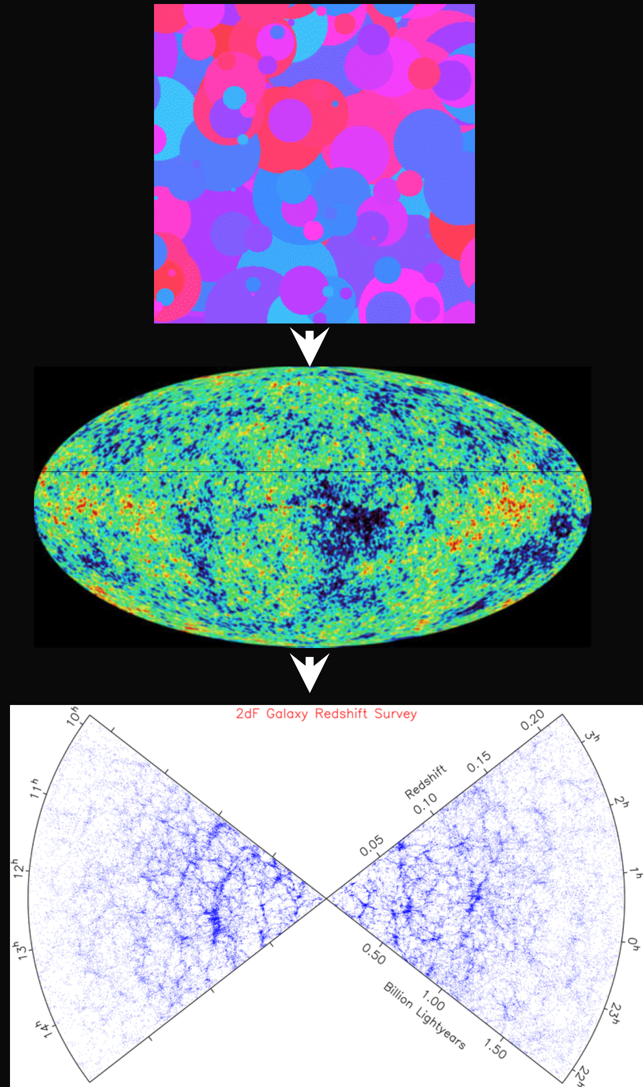
Inhomogeneities in the Universe

Relevant comments on inhomogeneities:

- Quantum fluctuations during inflation produce inhomogeneities
- CMB fluctuations ~ 1 in 100,000
- Galaxies today are clumped

Current thinking:

- Gravity amplifies fluctuations
- Before recombination
  - Competing effects of gravity and pressure
- Next lecture more in detail…
- Laws of physics predict what we see
  - Mainly gravity and electromagnetism



Power Spectrum of density fluctuations

Field of density fluctuations

$$\delta(x) = \frac{\delta\rho(x)}{\bar{\rho}}$$

Fourier transform

$$\delta(k) = \int d^3x\, e^{-ik\cdot x}\, \delta(x)$$

Power spectrum essentially square of Fourier transform

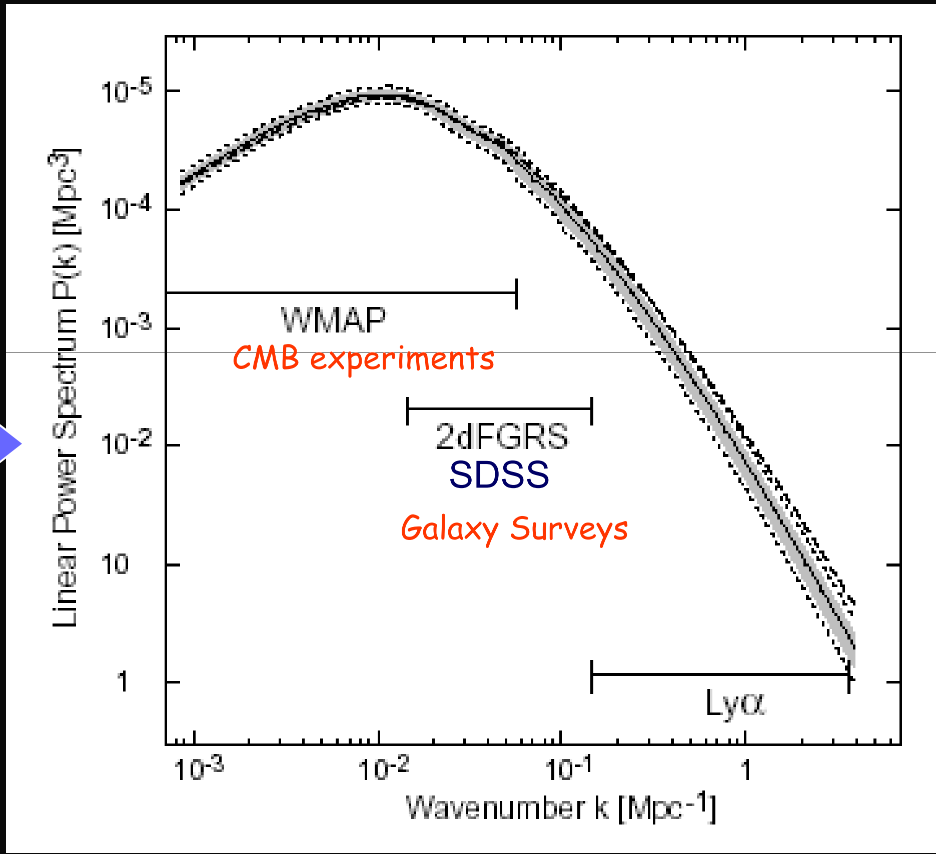
$$\langle \delta(k)\delta(k') \rangle = (2\pi)^3 \hat{\delta}(k-k')\, P(k)$$

with $\hat{\delta}$ the delta function

Power spectrum is Fourier transform of two-point correlation function

$$\xi(x) = \langle \delta(x_2)\delta(x_1) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} P(k)$$

where $x = x_2 - x_1$

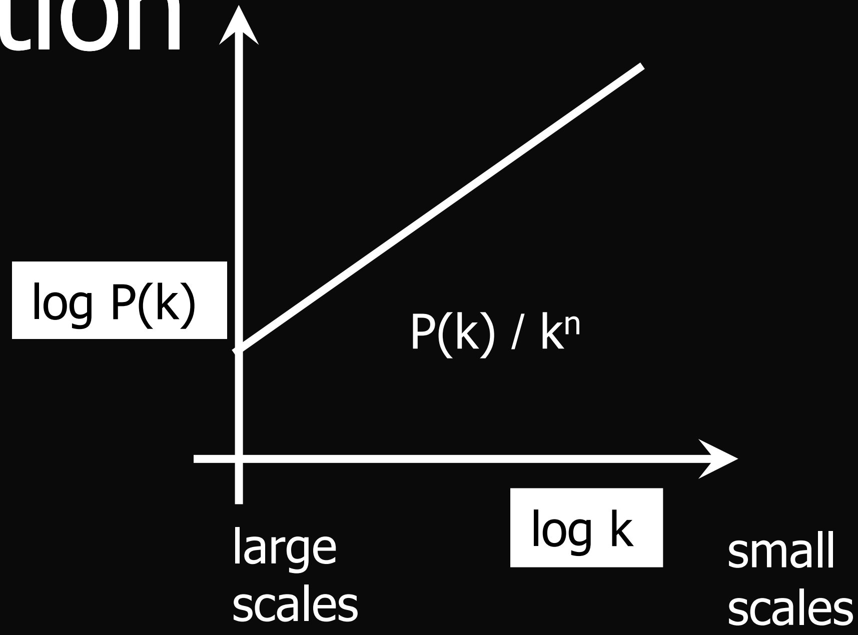


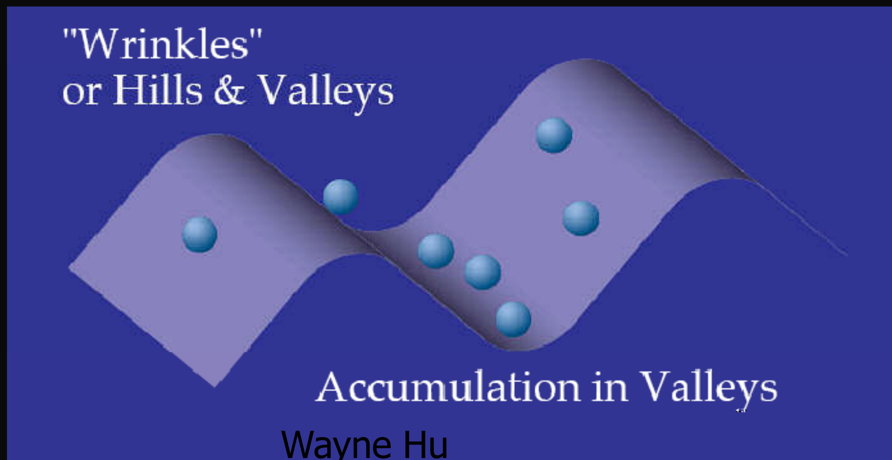
Physical understanding of the theoretical prediction

Ingredients

- Assumption about post-inflation P(k)
- Growth due to gravitational collapse
- Plasma oscillations
- We will see this in the next lectures…

- Inflation predicts $P(k) / k^n$
  - where $n \sim 1$
- Gravitational collapse amplifies fluctuations





Wayne Hu

http://background.uchicago.edu/~whu/beginners/introduction.html

Perturbations from Inflation:

- Quantum Fluctuations arise for this field with r.m.s. (not on syllabus, if interrested see QFT: Peacock) this is not an obvious result arising from the equations for a scalar filed $\delta\phi \simeq H/2\pi$
- Parts of the Universe with different ϕ exit inflation at different times, differing by $\delta t = \delta\phi/\dot{\phi}$
- Hence there is a fractional variation in energy densities on the Hubble radius scale of $\delta_H \simeq H\delta t = \frac{H\delta\phi}{\dot{\phi}} = \frac{H^2}{2\pi\dot{\phi}}$
- While V ≈ constant, H ≈ constant, so horizon-scale perturbations are roughly constant. This gives *scale invariant* fluctuations $P(k)k^3 \propto \Delta^2(k) \propto \delta_H^2$
- Fluctuations of a certain wave number *k cross the Hubble* radius when $k^{-1} = 1/(aH)$.
- Power spectrum of density fluctuations (this is on the syllabus) $P(k) \propto k^n;\ n \simeq 1$

END for now!!!