Non-parametric foreground fitting

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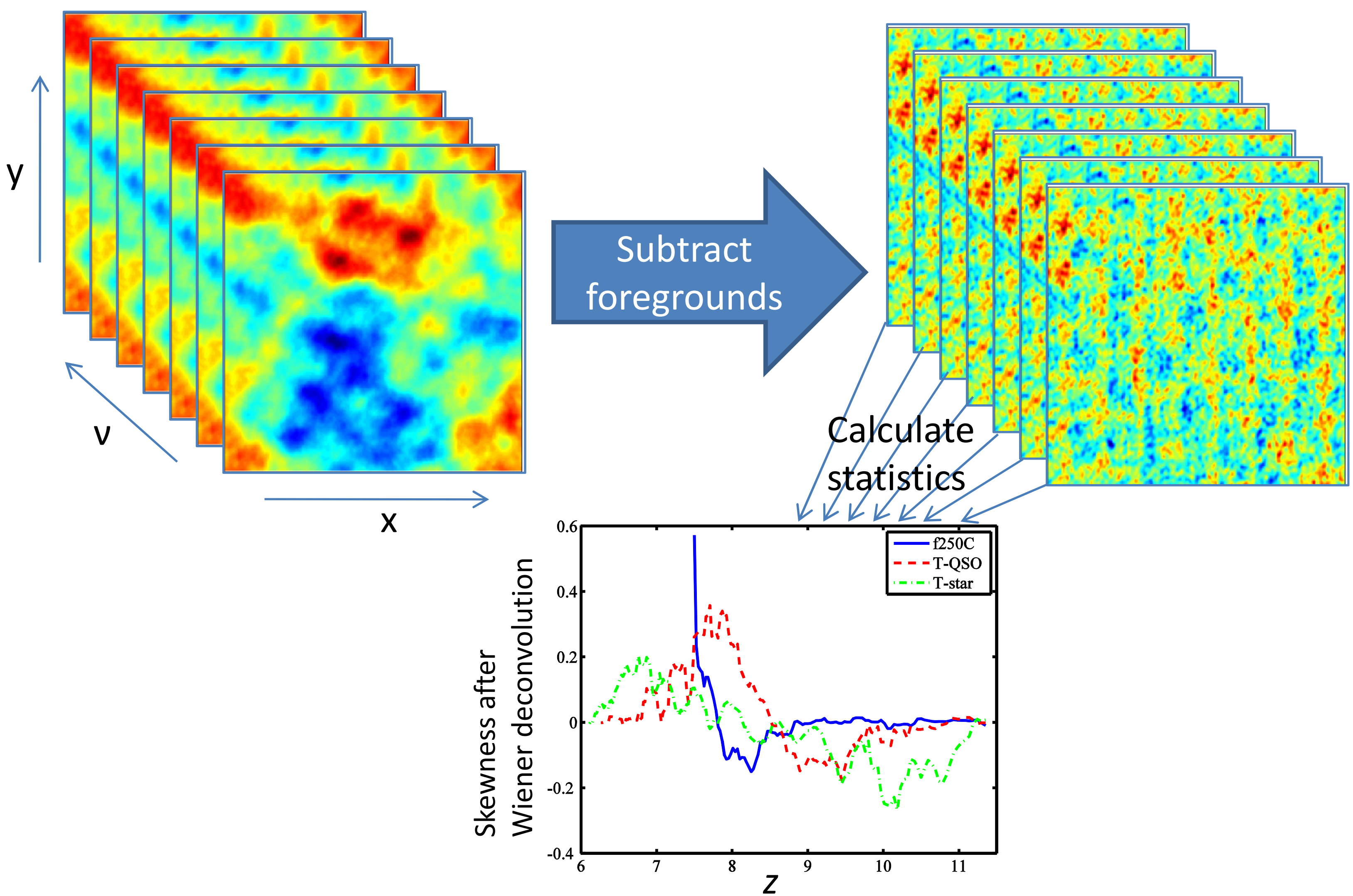
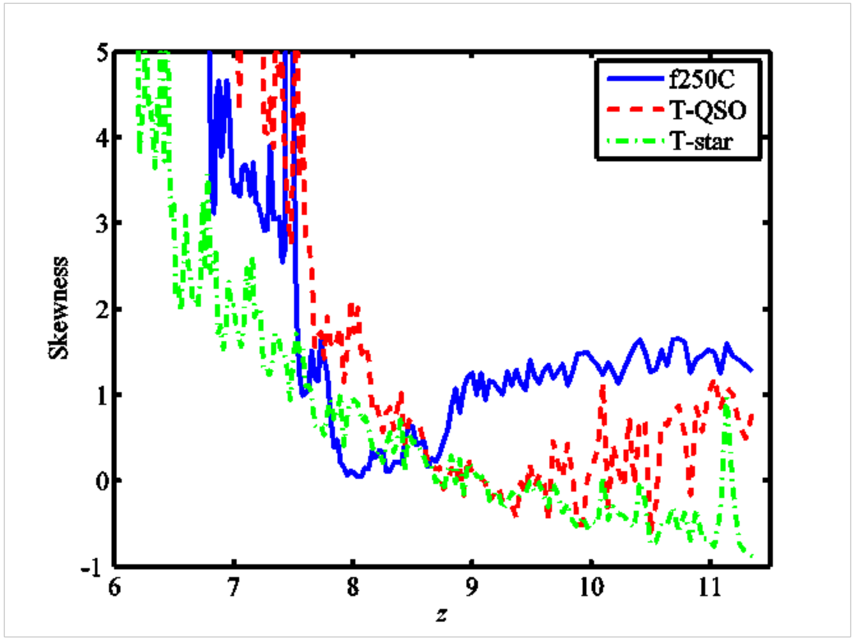
Signal extraction in two stages

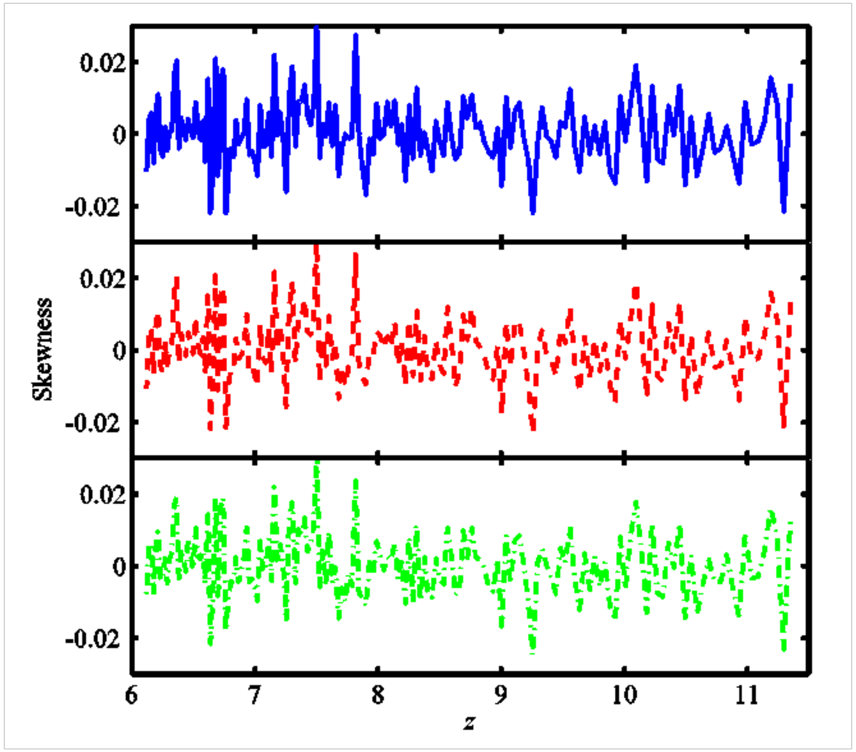


The importance of good foreground fitting

Original simulations



Residuals



The importance of good foreground fitting

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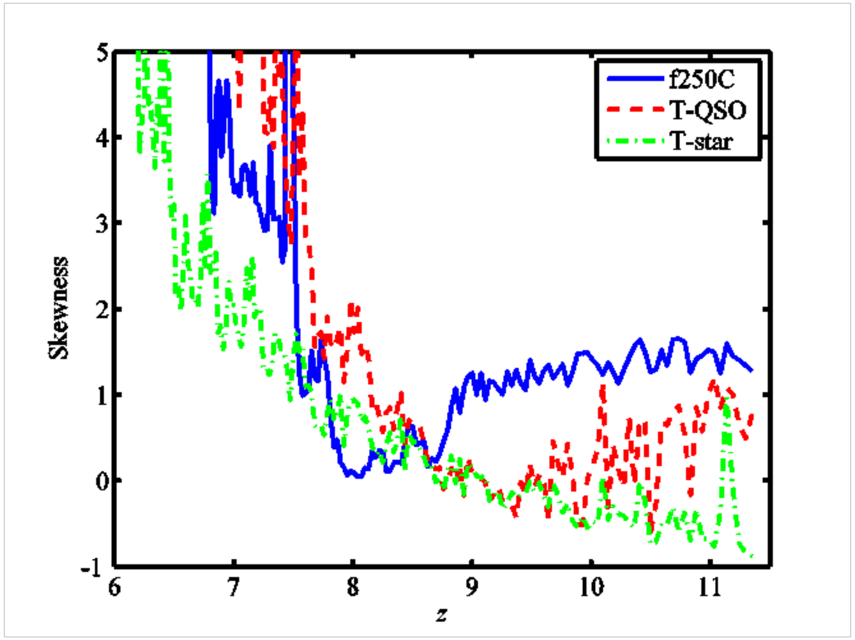
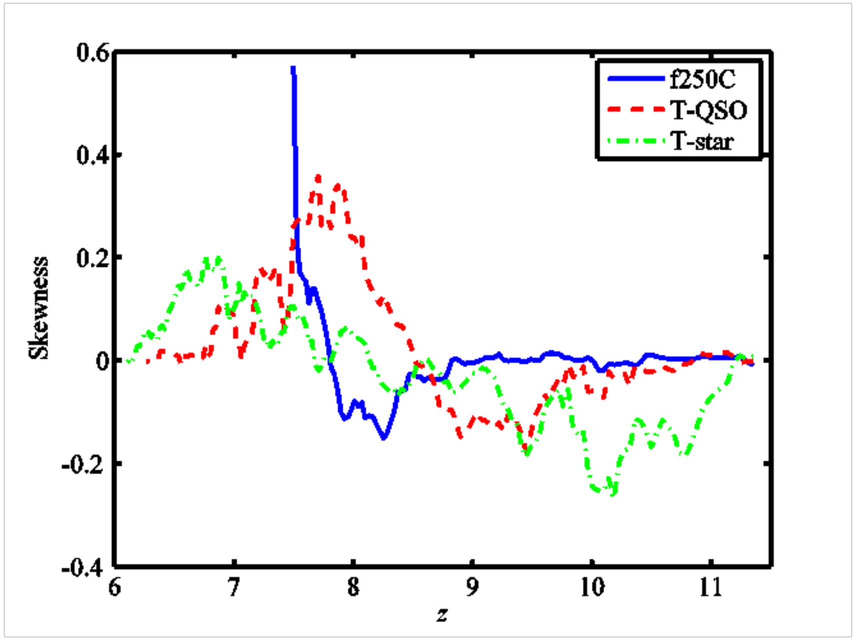


Residuals after Wiener deconvolution



The importance of good foreground fitting

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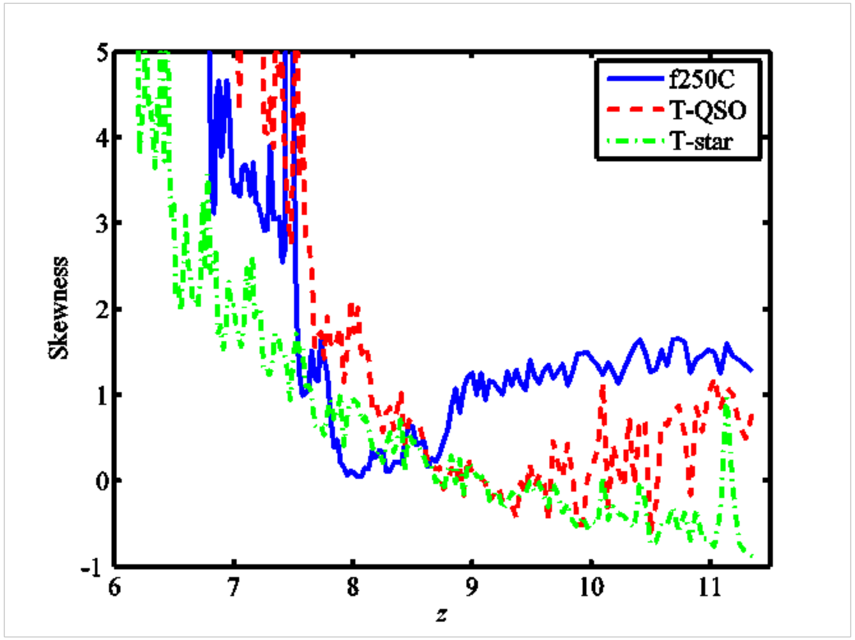
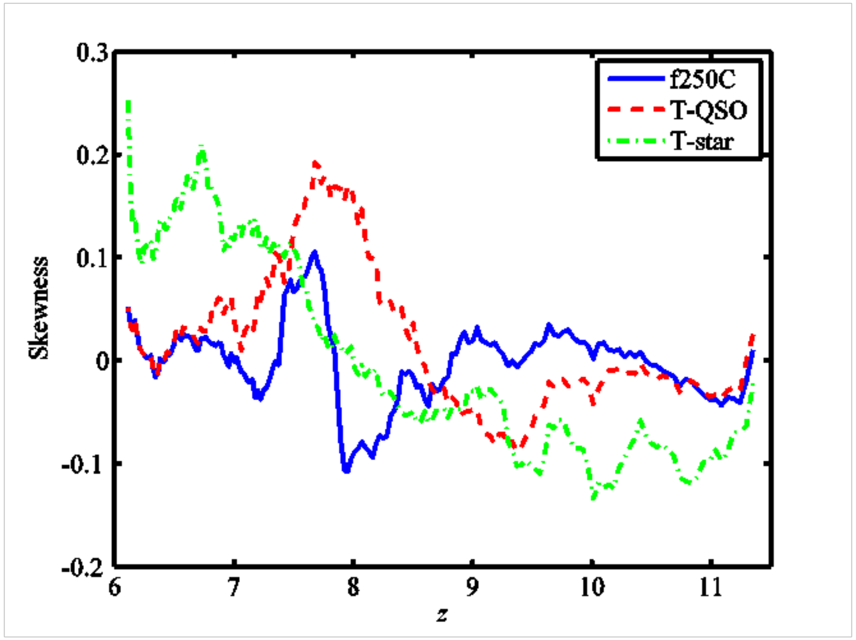


Residuals after perfect foreground subtraction



Wish list for a foreground fitting algorithm

- Accuracy.
- Lack of bias.
- Avoidance of under-fitting or over-fitting.
- Make minimal assumptions about the functional form of the foregrounds; i.e., exploit their smoothness directly.
- Speed (less important if we only wish to subtract the foregrounds once, in post-processing).

Statistical approach

- Model data points (x_i ,y_i) by:

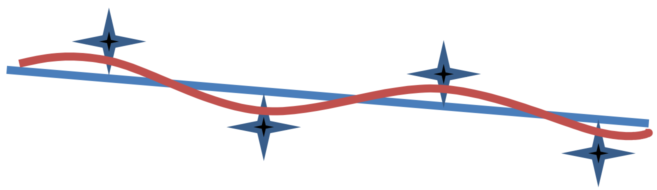
$$y_i = f(x_i) + \varepsilon_i, \;\; i = 1, \ldots, n$$

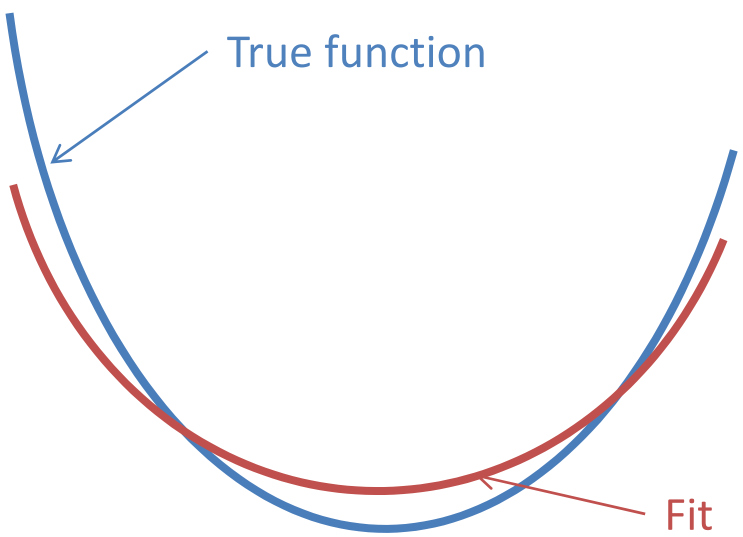
- Then we wish to solve the following problem:

$$\min_f \left\{ \underbrace{\sum_{i=1}^{n} \rho_i(y_i - f(x_i))}_{\text{“Least squares”}} + \underbrace{\lambda R[f]}_{\text{Roughness penalty}} \right\}$$

Choosing a roughness penalty *R*[*f*]

- Require a roughness penalty that stops the curve wiggling towards individual data points, but avoids the problem of attrition.
- 'Smoothing splines' use integrated curvature as the roughness penalty, but in Wp smoothing the integrated *change* of curvature is used instead.





Wp smoothing

- An approximation to the change of curvature, *f'''/f''*, blows up at the inflection points *f''*=0.
- *R*[*f*] measures the change of curvature 'apart from the inflection points', w_i
- Perform the minimization with the position of the inflection points (and s_f) fixed.

$$R[f] = \int_{x_1}^{x_n} h'_f(t)\mathrm{d}t$$

$$f''(x) = p_{\mathbf{w}}(x)e^{h_f(x)}$$

$$\begin{aligned} p_{\mathbf{w}}(x) &= s_f(x-w_1)(x-w_2) \\ &\quad \times \ldots (x-w_{n_w}) \end{aligned}$$

Wp smoothing

- Mächler (1993,1995), who proposed the method, showed that the variational problem leads to the following differential equation:

$$h_f'' = p_{\mathbf{w}} e^{h_f} \left[-\frac{1}{2\lambda} \sum_{i=1}^{n} (x - x_i)_+ \psi_i(y_i - f(x_i)) \right]$$

where a_+ =max(0,a), $\psi_i(\delta) = \frac{\mathrm{d}}{\mathrm{d}\delta}\rho_i(\delta)$, and the boundary conditions are

$$h_f'(x_1) = h_f'(x_n) = \sum_i \psi_i(y_i - f(x_i)) = \sum_i x_i \psi_i(y_i - f(x_i)) = 0$$

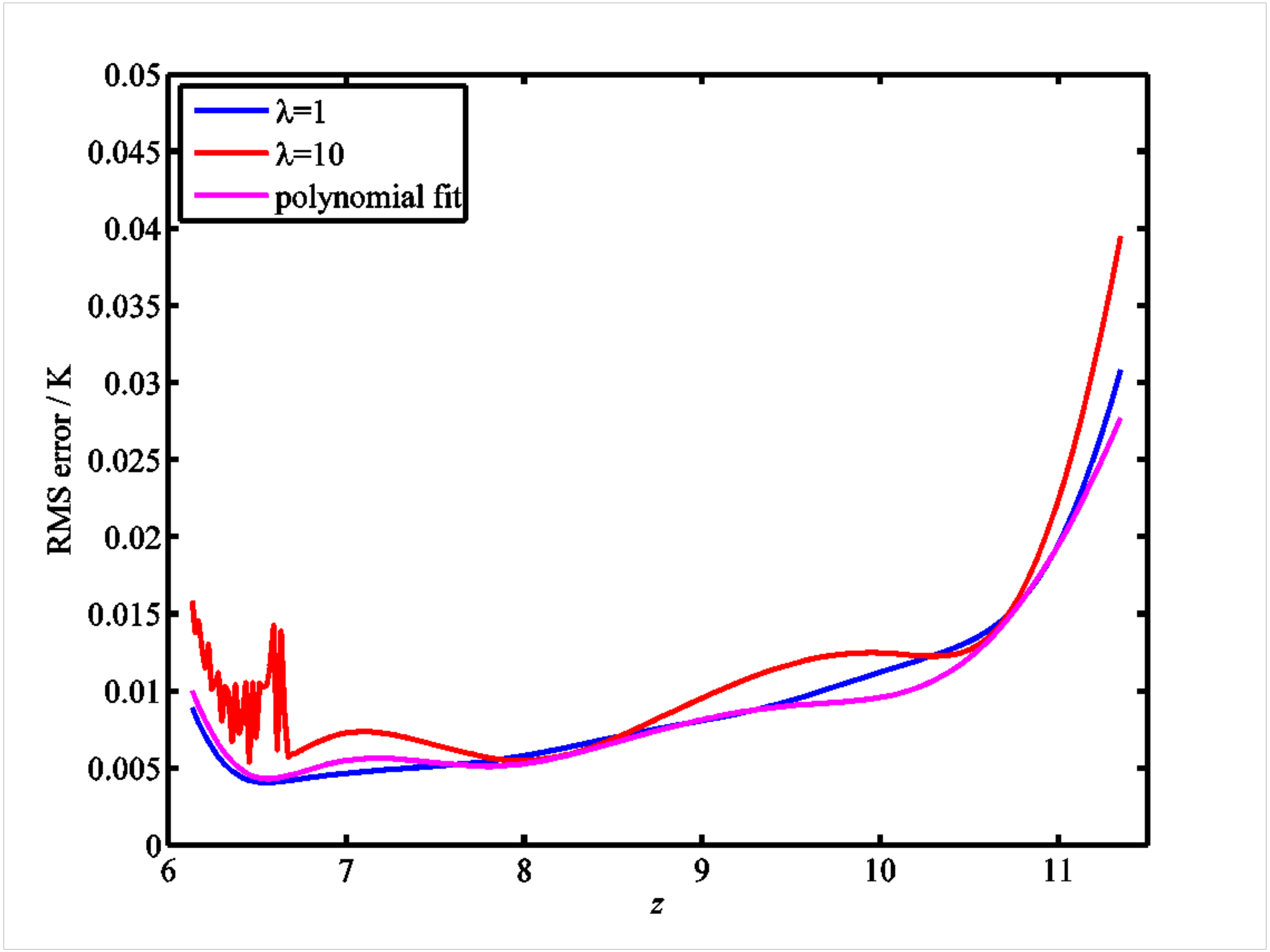
Implementation

- In general we need a method to find the number of inflection points, and need to perform a further minimization over their position.
- For the foreground fitting we find that it works well to have no inflection points (this would be the case anyway for a sum of negative-index power laws).
- The differential equation and the boundary conditions are in a nonstandard form:
  - Can rewrite as a system of 5*n*-4 coupled first-order equations and use a standard BVP solver.
  - Alternatively, convert to a finite difference equation and perform a multidimensional function minimization (seems better so far).
- Either approach requires a reasonable initial guess for the solution; we fit a power law since this has no inflection points.

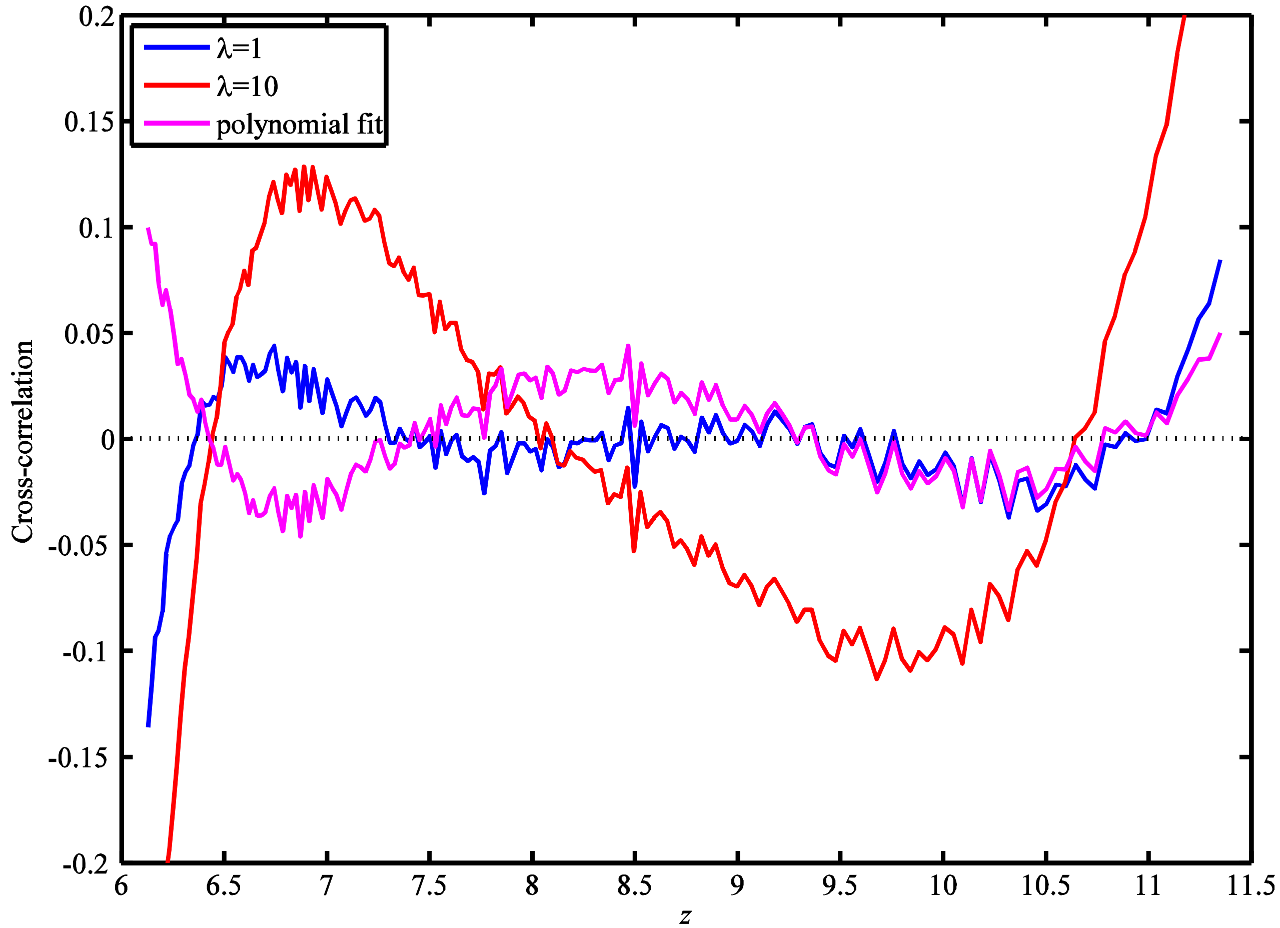
Results

- Approx. 3s of computing time per sightline for 170 points; this depends on the quality of the initial guess.
- *rms* fitting errors small compared to the random noise and comparable to or better than for polynomial or power law fitting (where we have to have assumed a functional form).
- Better cross-correlation properties with the (known, simulated) foregrounds compared to polynomial fitting.

RMS fitting error



Cross-correlation of residuals with foregrounds



Ongoing work

- Find the best value for λ.
- What's the effect of using more or fewer bins?
- Ways to alleviate the problems at the ends of the range (change weighting scheme?); can we deal with gaps?
- Generalize and speed up the Wp algorithm (another use for GPUs?).
- Does the improved foreground fitting allow us to relax the assumptions we make when processing the foreground-subtracted images (e.g. the signal correlation matrix in Wiener deconvolution)?
- Power spectrum estimation; discriminating between models.
- Other statistics.

Conclusions

- Accurate and unbiased foreground fitting is a crucial part of our signal extraction.
- Non-parametric methods do not require us to specify a particular functional form for the foregrounds.
- Wp smoothing, which penalizes the integrated change of curvature (apart from inflection points) is a promising method.
- Implementations are computationally expensive at the moment but not unreasonable.
- We find it gives accurate and unbiased estimates of the simulated foregrounds making only general assumptions about smoothness, especially in the middle of the frequency range.