Progress on power spectrum extraction

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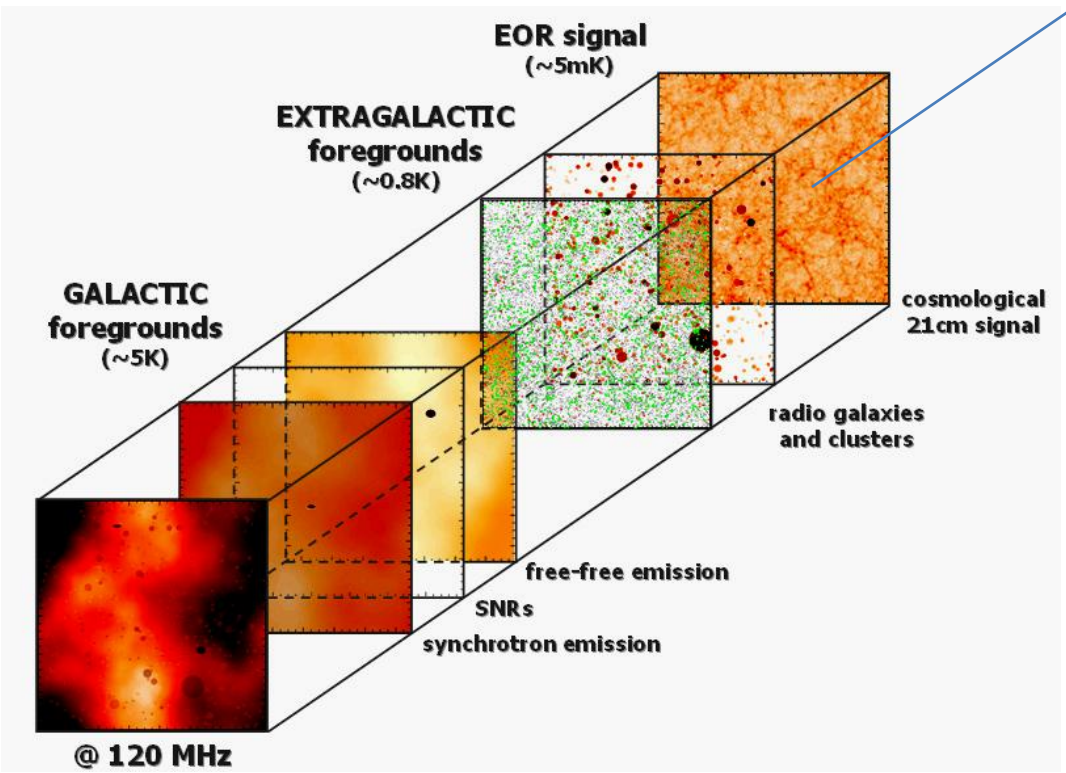
Motivation (I)
Information from the power spectrum

$$\frac{\delta T_{\rm b}}{\rm mK} = 39h(1+\delta)x_{\rm HI}\left(1-\frac{T_{\rm CMB}}{T_{\rm spin}}\right)\left(\frac{\Omega_{\rm b}}{0.042}\right)\left[\left(\frac{0.24}{\Omega_{\rm m}}\right)\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}$$

- Measure the power spectrum of $\Delta\delta T_{\rm b}$, the difference of $\delta T_{\rm b}$ from the mean.
- Contains information on:
  - the growth of structure (through 1+δ);
  - reionization (through *x*), e.g. growth of bubbles;
  - heating (through dependence on the spin temperature);
  - cosmology;
  - redshift-space distortions.
- Decomposing $P(k,\mu)$ in powers of μ may help disentangle these effects.

Components of the data cubes



Foregrounds as in Jelić et al. (2008).

- Cosmological signal here from the f250C simulation of Iliev et al. (2008).
- Foregrounds and cosmological signal are convolved with the instrumental response.
- Uncorrelated noise in the *uv* plane: for one year with one beam, corresponds to an *rms* of 52mK at 150 MHz.
- Need foregrounds which are smooth as a function of frequency.

Wp smoothing: non-parametric foreground fitting

- Model data points (x_i ,y_i) by:

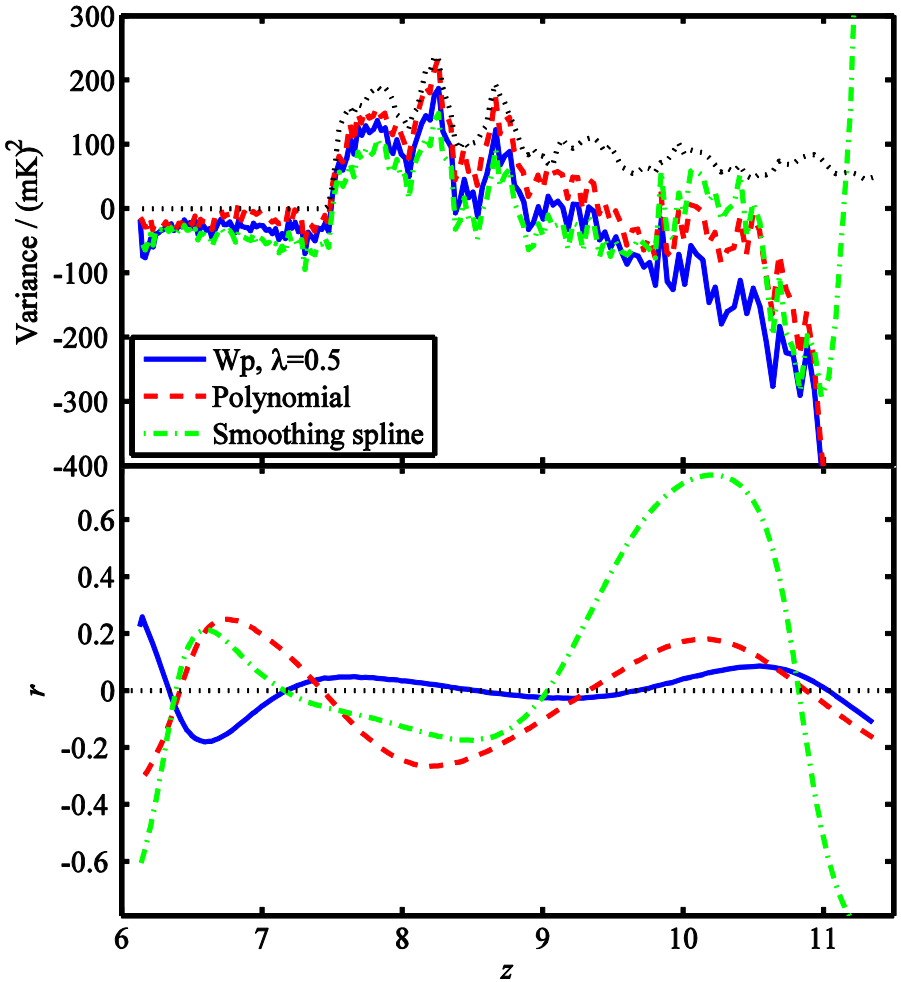
$$y_i = f(x_i) + \varepsilon_i, \;\; i = 1, \ldots, n$$

- Then we wish to solve the following problem:

$$\min_f \left\{ \underbrace{\sum_{i=1}^{n} \rho_i(y_i - f(x_i))}_{\text{“Least squares”}} + \underbrace{\lambda R[f]}_{\text{Roughness penalty}} \right\}$$

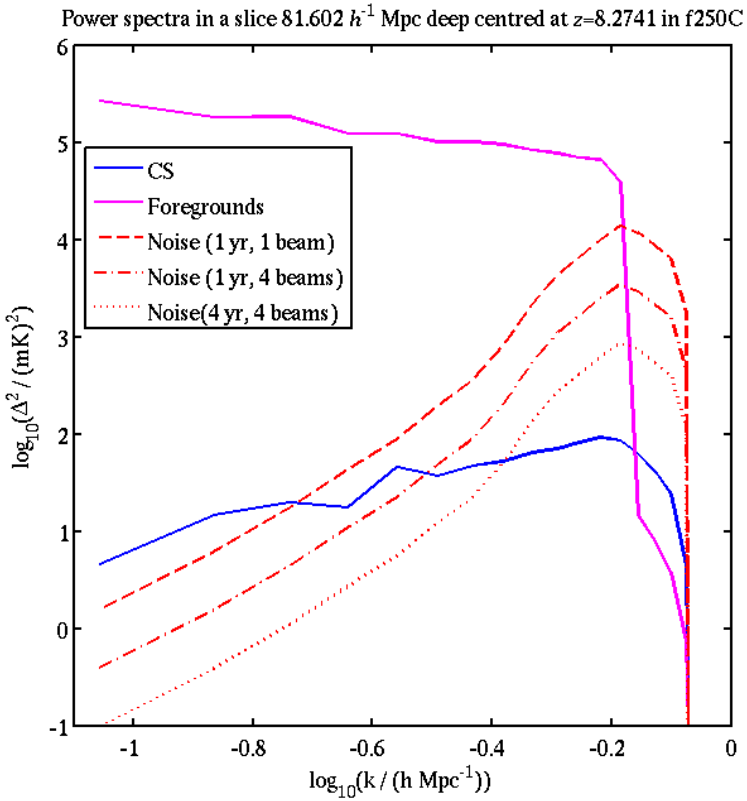
- Here the roughness penalty measures the integrated change in curvature ‘apart from inflection points’; inflection points are the primary measure of roughness.
- The solution of this minimization is the solution of a boundary value problem derived by Mächler.
- ‘Wp’ stands for ‘Wendepunkt’.

Motivation (II)
Step 1: Measure the RMS?



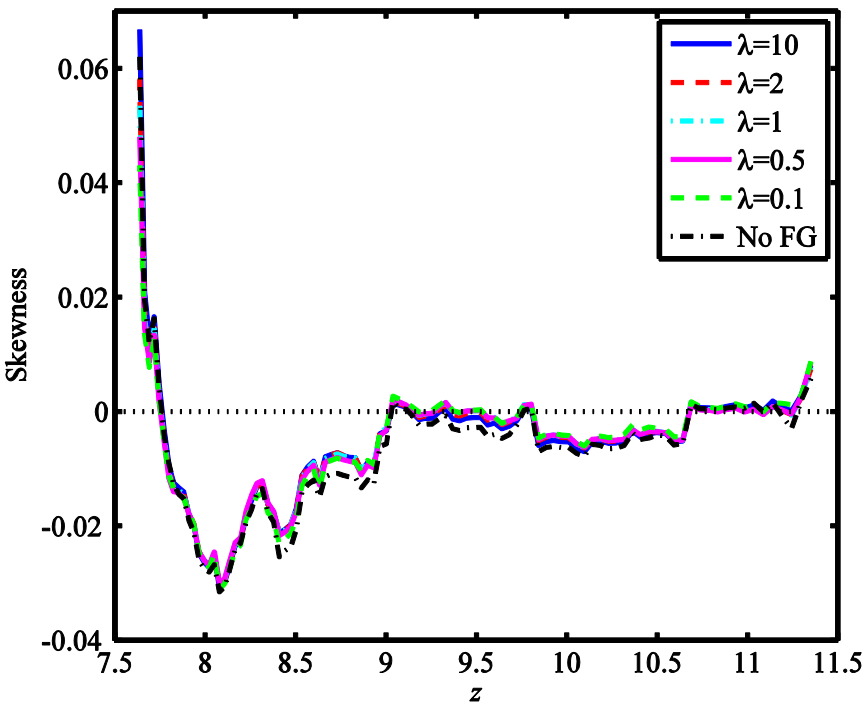
- First aim: establish that we can detect any signal at all from the EoR.
- Seems reasonable to do this using the integrated RMS, which evolves with redshift.
- Unfortunately, polluted by under- and over-fitting, and by edge effects.
- Can using scale-dependence help?

Scale dependence and size of components of the signal



- Noise (receiver noise plus sky noise) dominates on small scales, leading to problems from over-fitting.
- Foregrounds dominate on larger scales, leading to problems from under-fitting.
- All scales contribute to the integrated RMS, but using the whole power spectrum we may be able to pick out the most favourable scales.
- Recovered shape provides a further check.

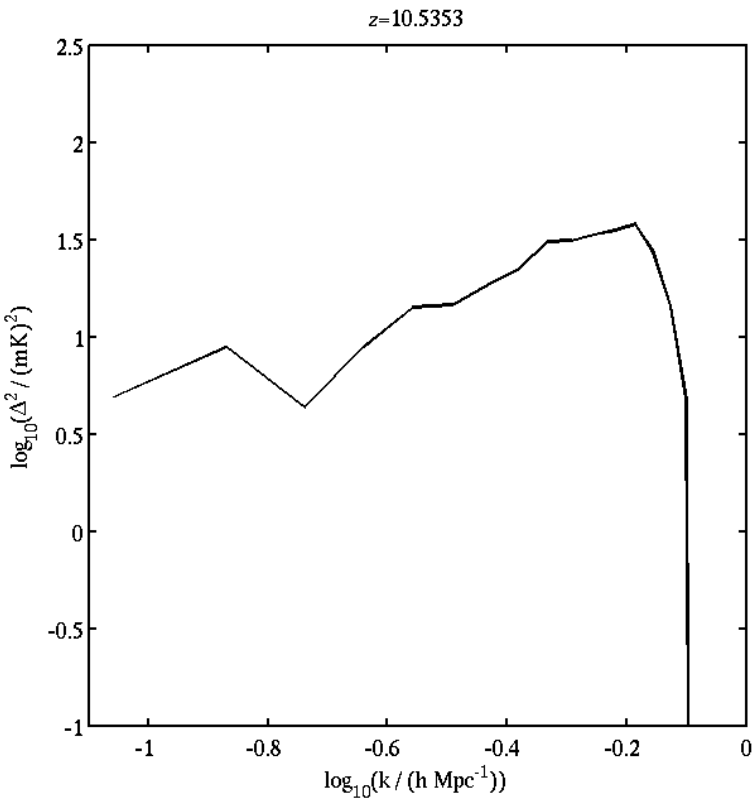
Motivation (III): using the power spectrum to recover the skewness



- Recovery seems to be robust to changes in the fitting algorithm.

- Redshift evolution of the skewness of residual maps may provide a confirmation of a detection (especially if we see negative skewness).
- Need to deconvolve the maps (e.g. Wiener deconvolution) before calculating the skewness, which requires an estimate of the correlation matrix (and hence angular power spectrum) of the signal.

Example evolution of the signal power spectrum



- Spectra here are convolved with the instrumental response (hence high-*k* cut-off).
- Growth of a feature on small scales due to formation of bubbles.
- Feature broadens and moves to larger scales as bubbles grow.
- At low redshift, signal drops because of a low neutral fraction.

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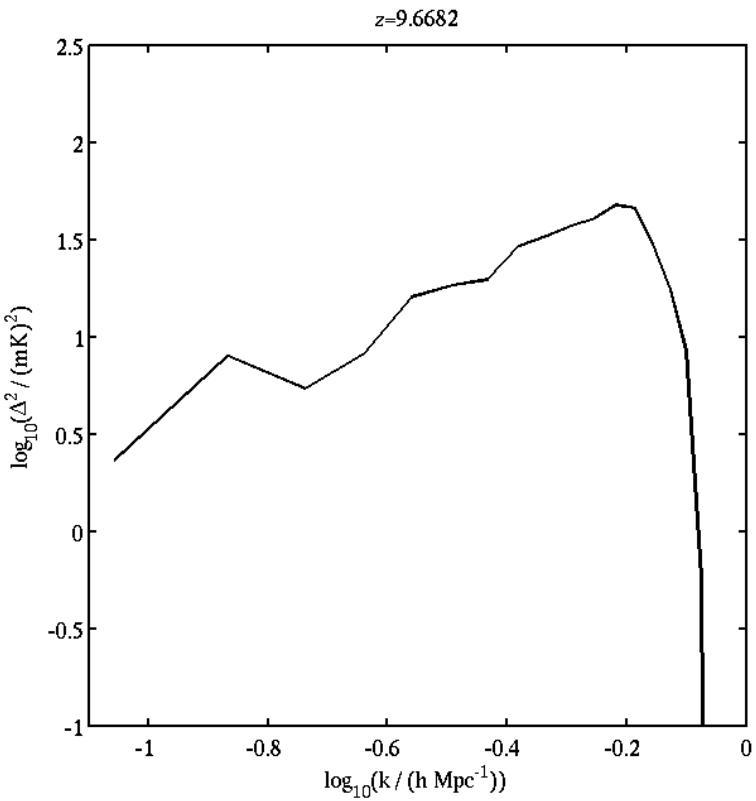

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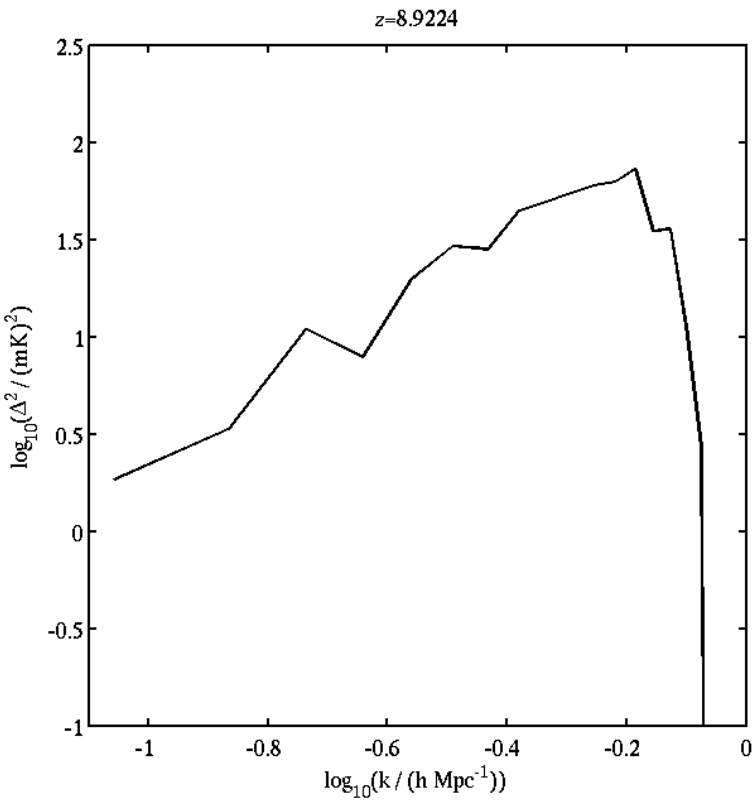

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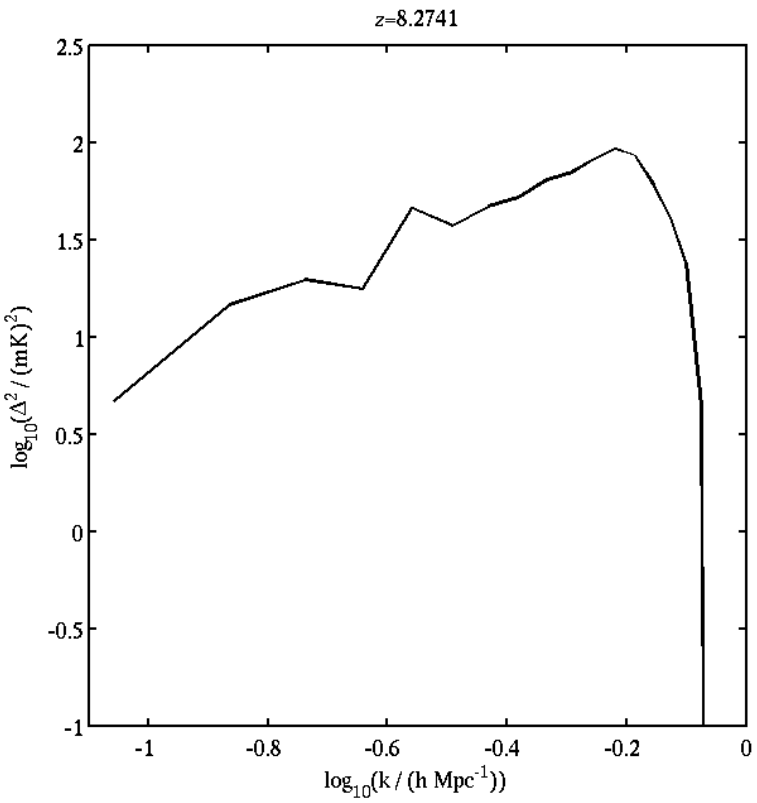

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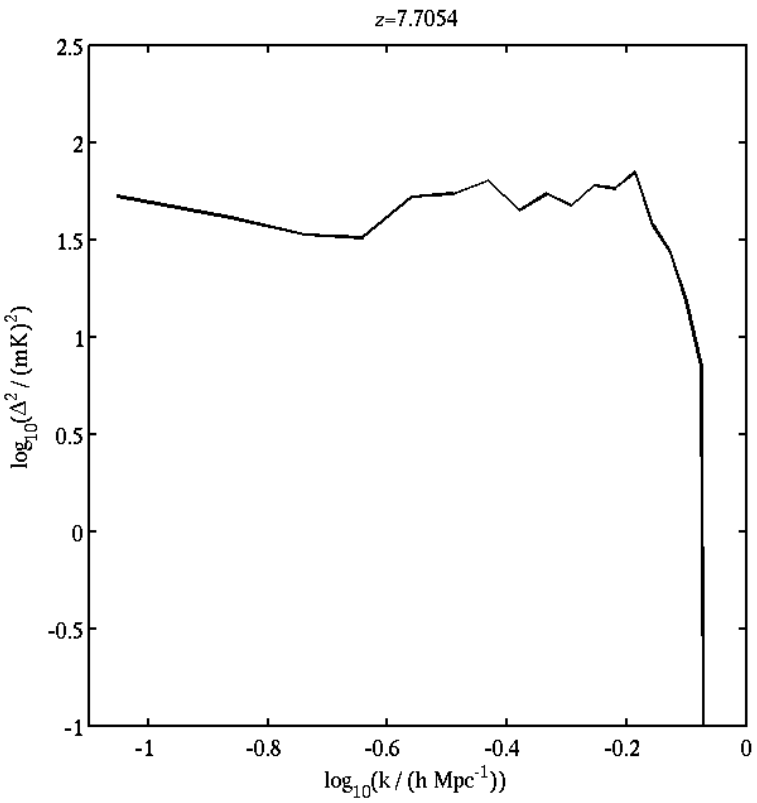

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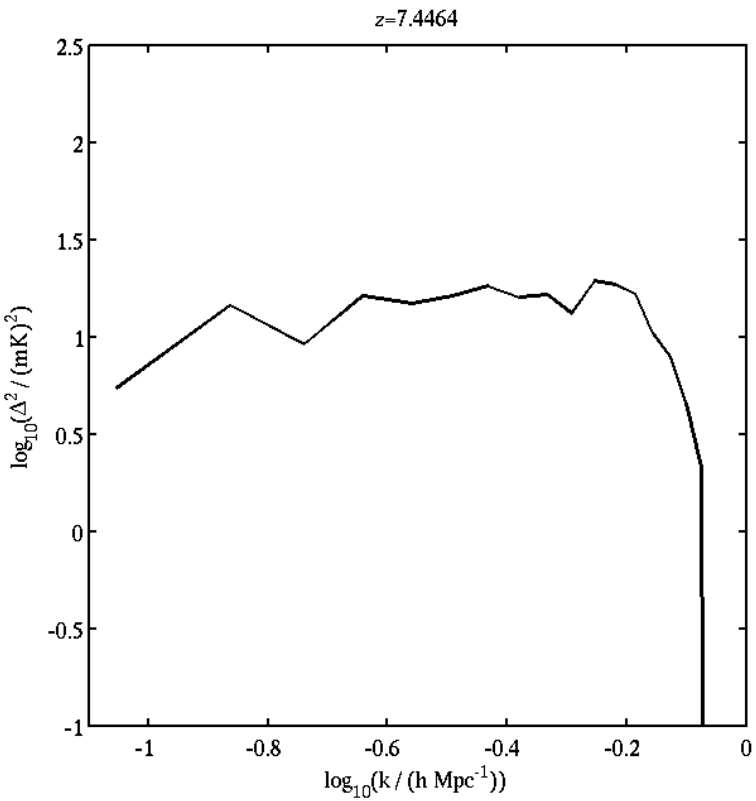

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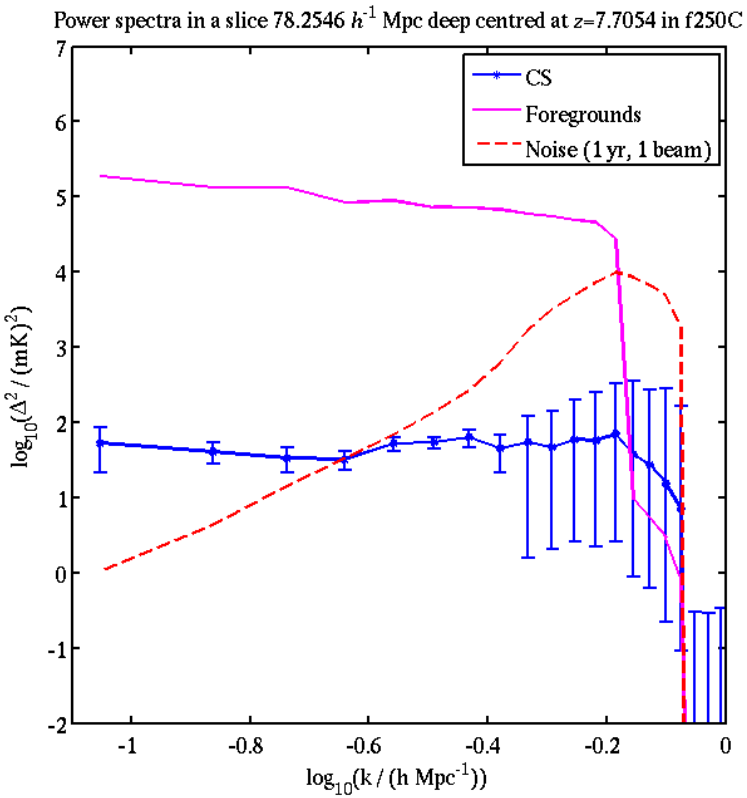
Power spectra with perfect foreground subtraction (1 year, 1 beam)

Low redshift



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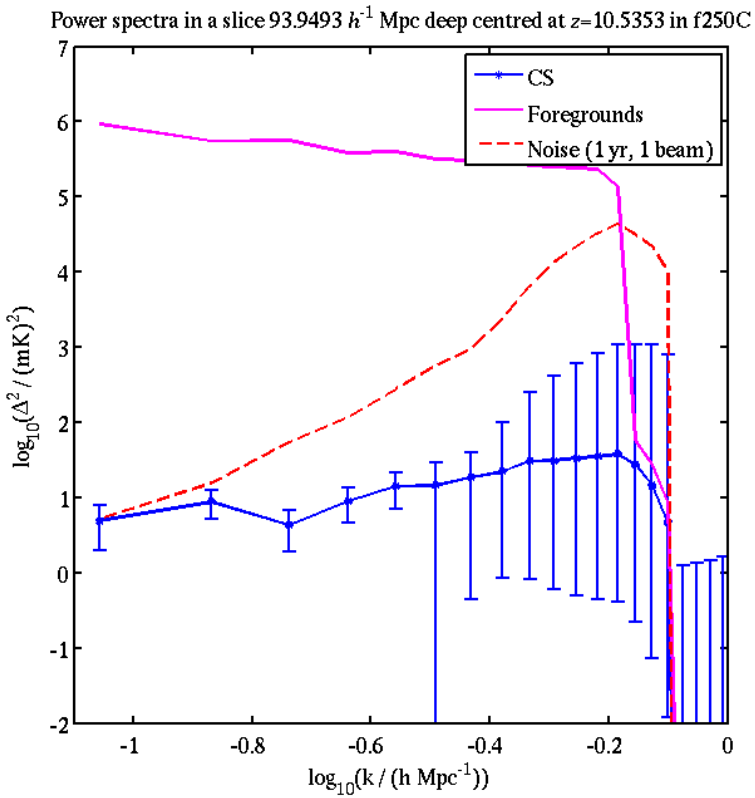

Power spectra with perfect foreground subtraction (1 year, 4 beams)

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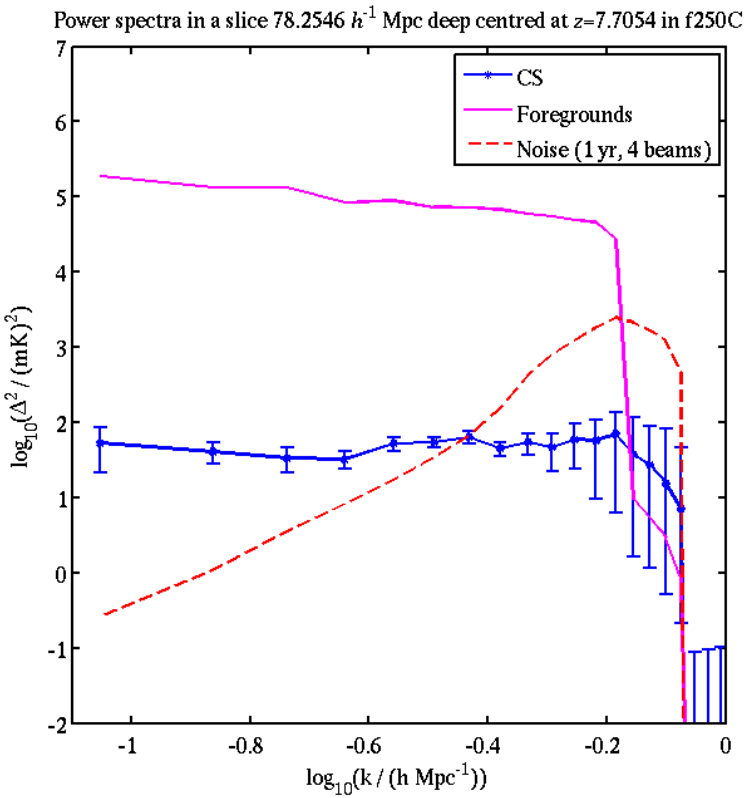

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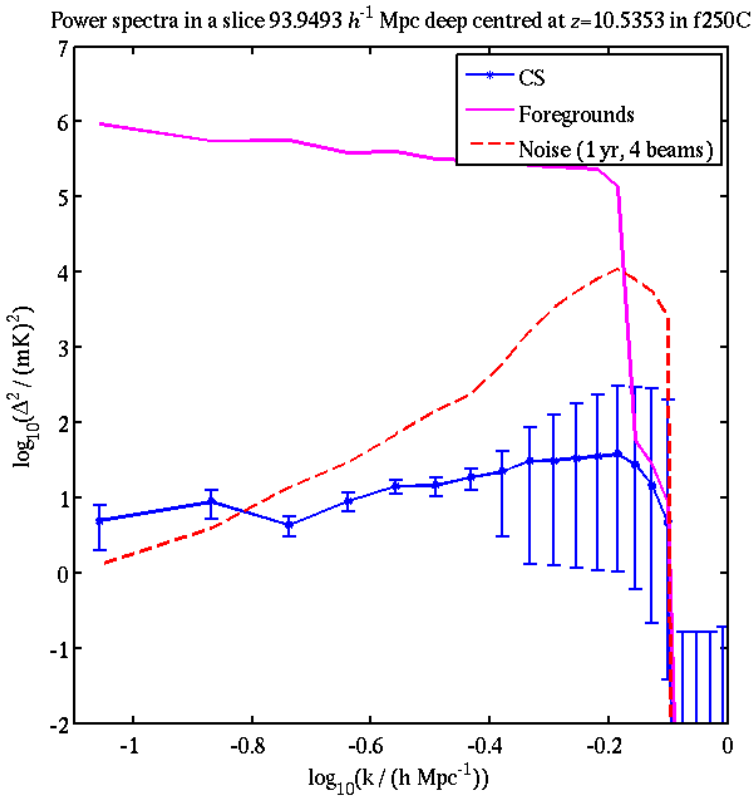

Power spectra with perfect foreground subtraction (4 years, 4 beams)

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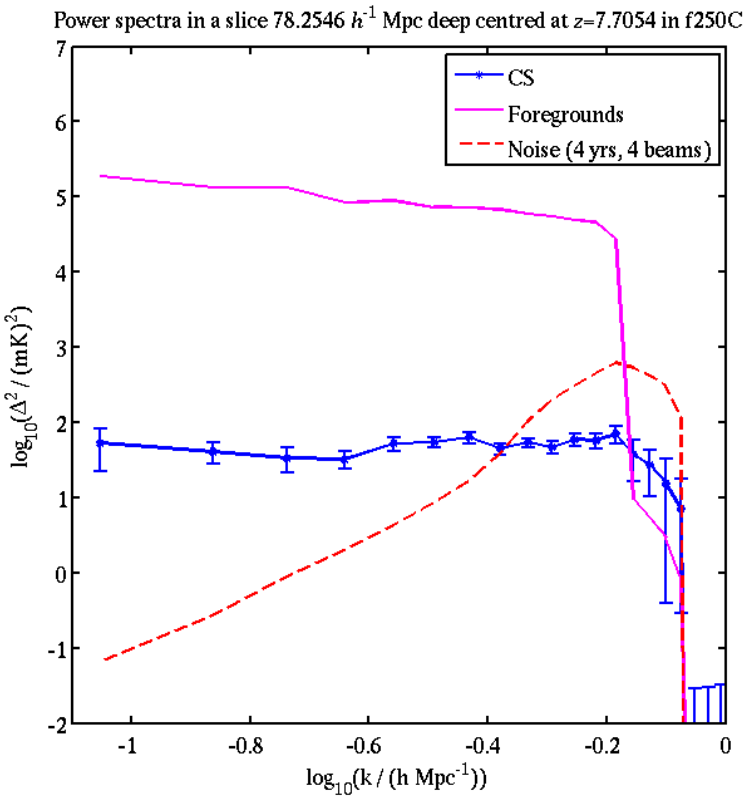

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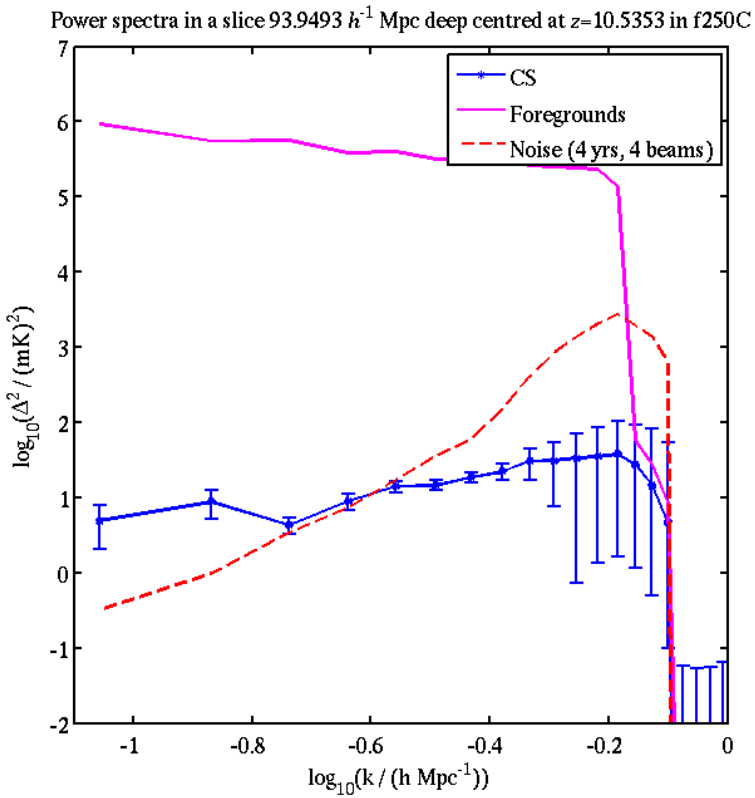
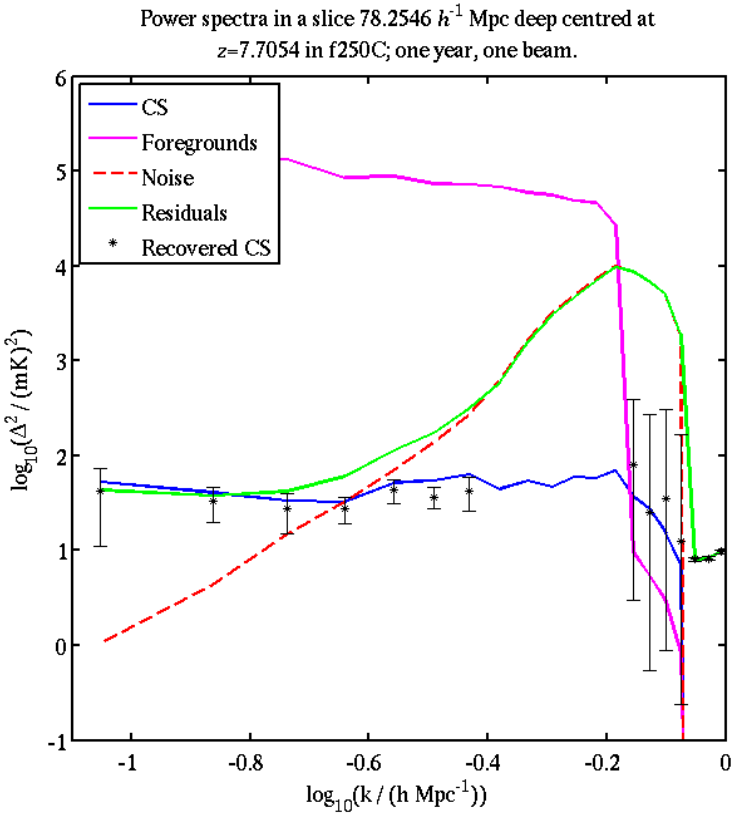


Results using Wp smoothing for foreground subtraction (1 yr, 1 beam)

Low redshift



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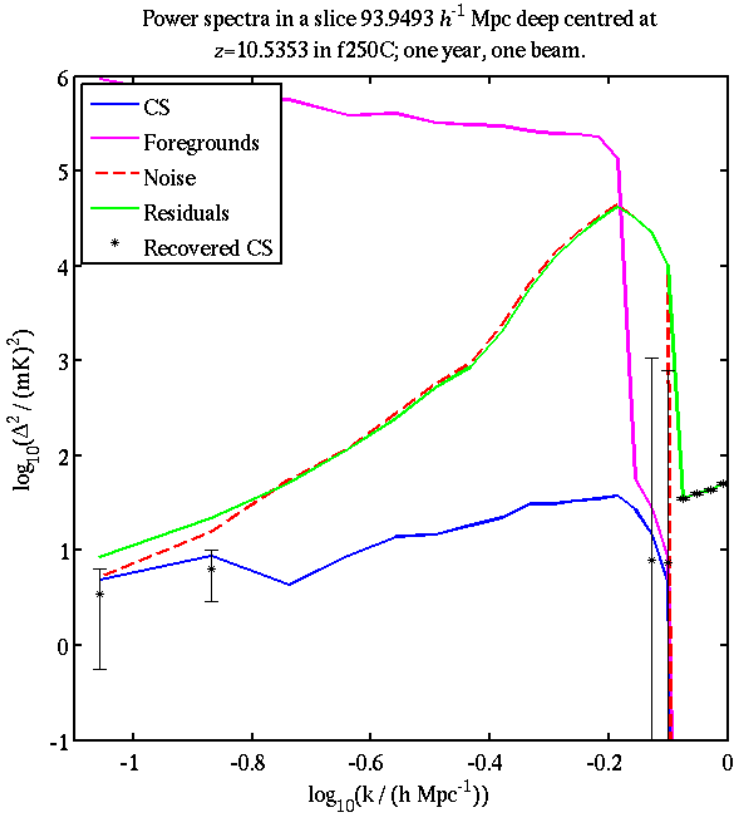

Results using Wp smoothing for foreground subtraction (1 yr, 4 beams)

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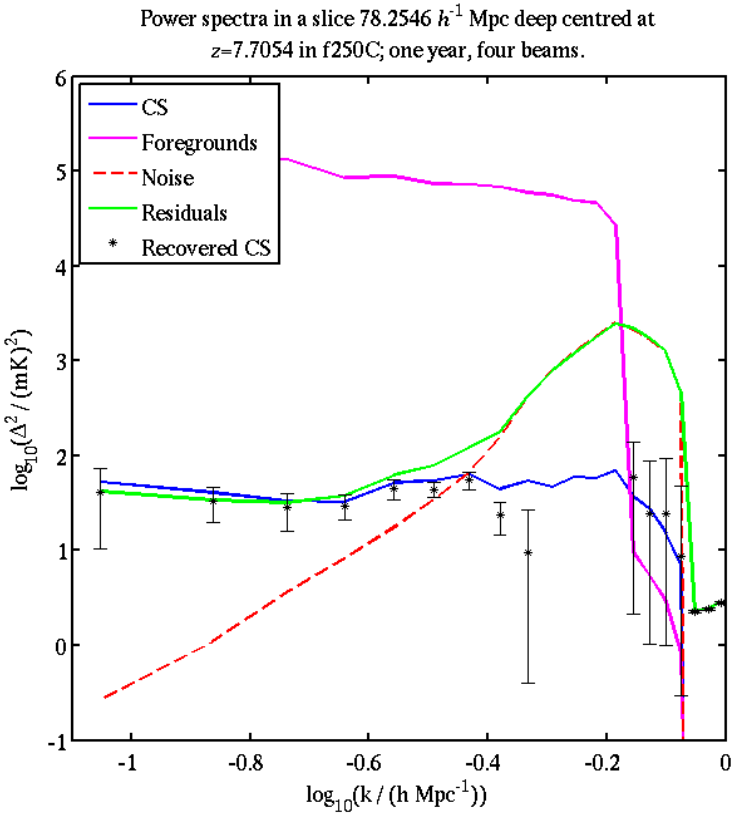

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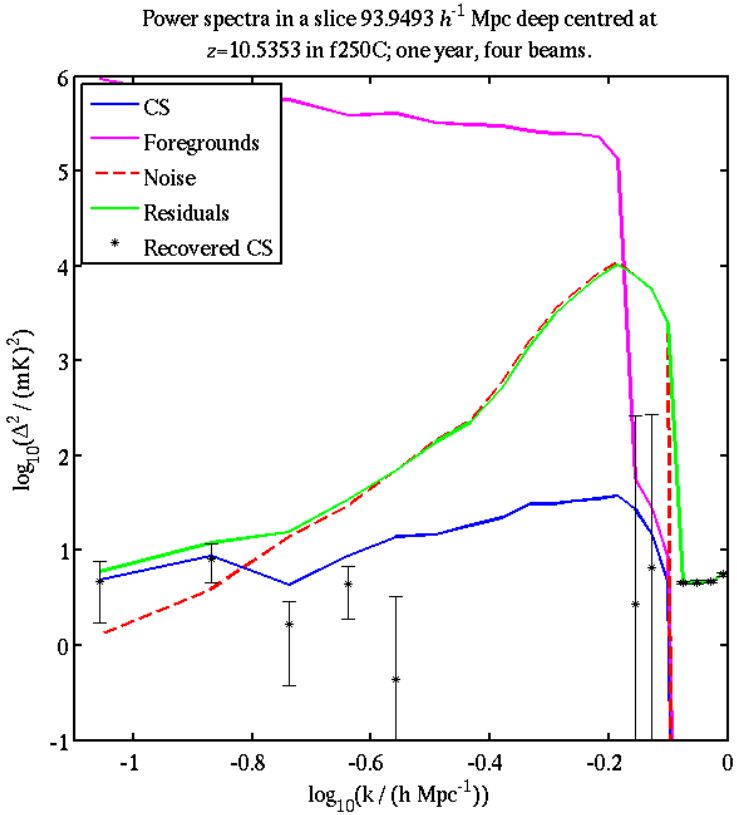
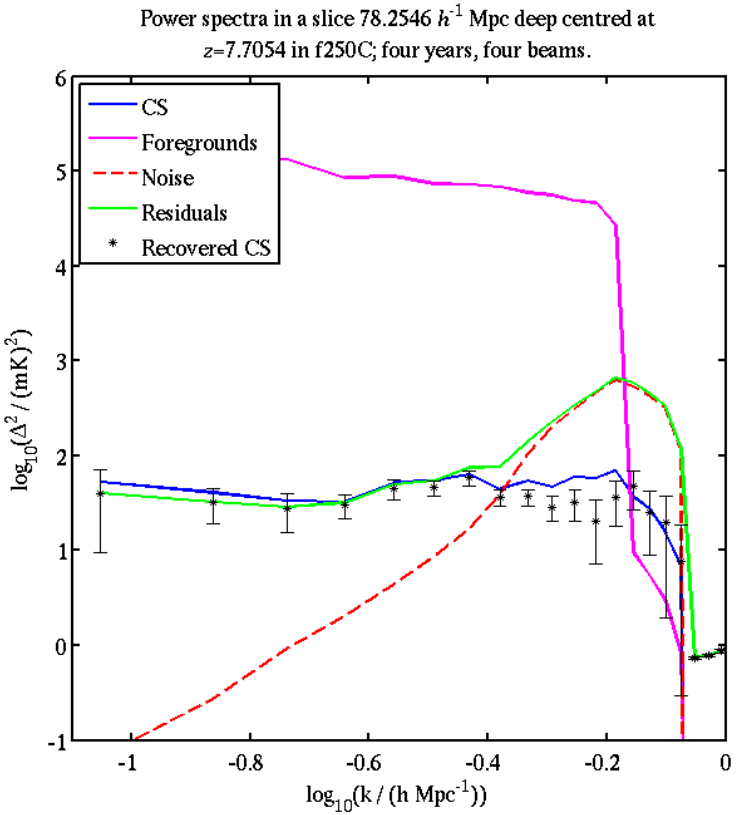


Results using Wp smoothing for foreground subtraction (4 yr, 4 beams)

Low redshift



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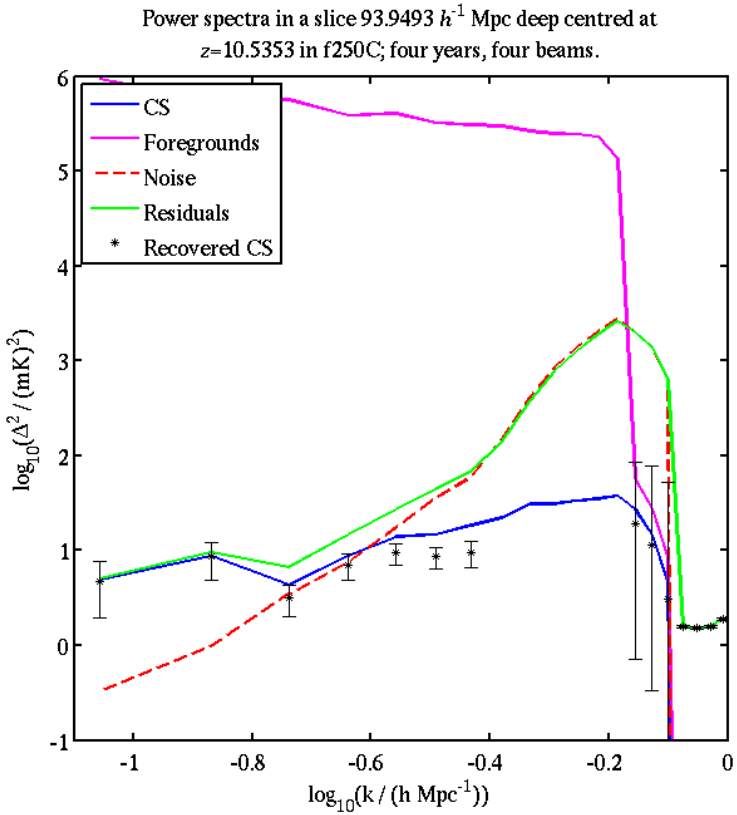
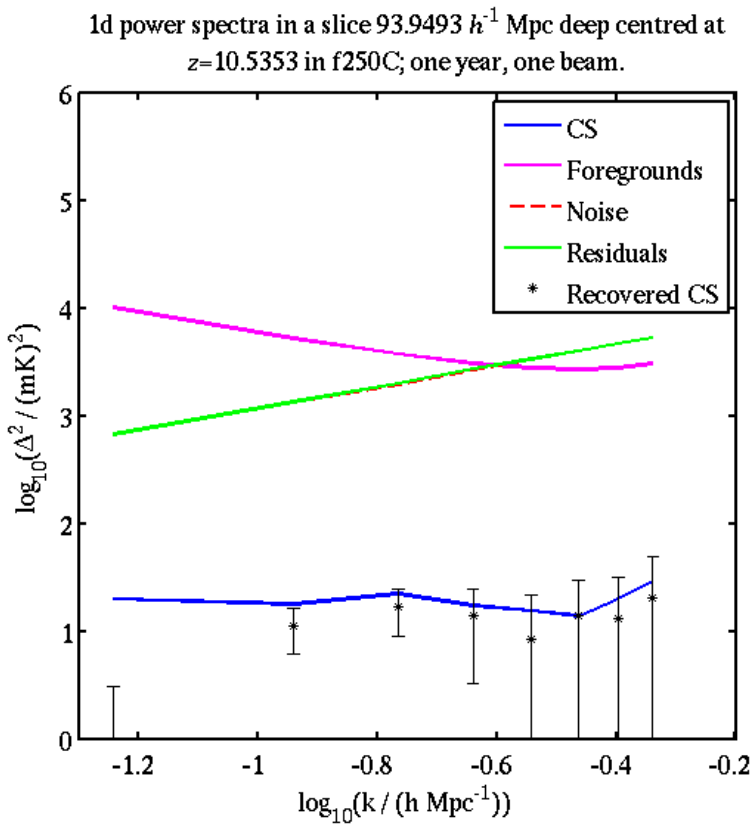
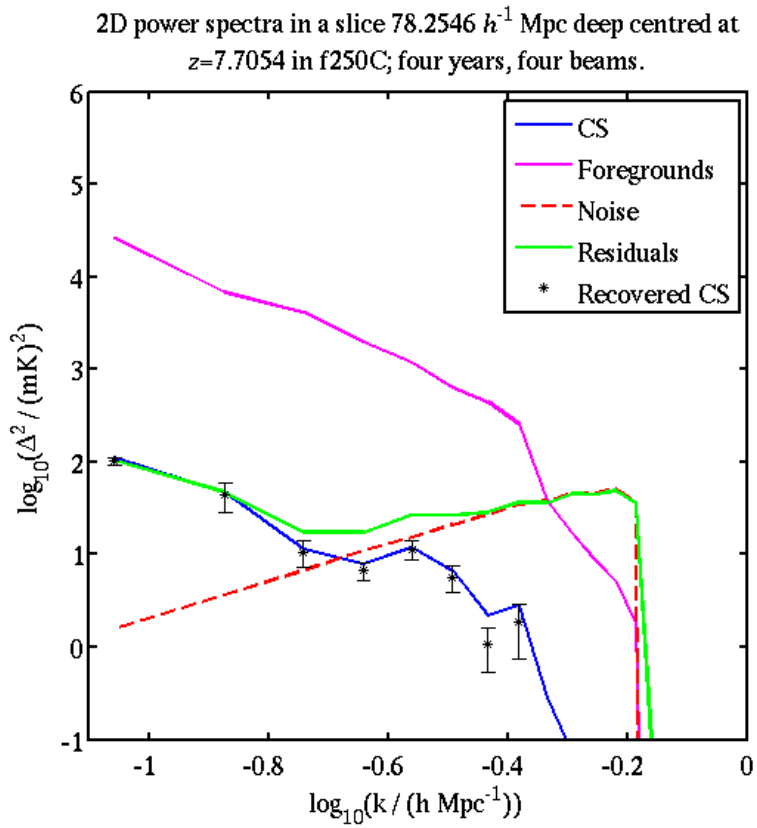


1D and 2D power spectra

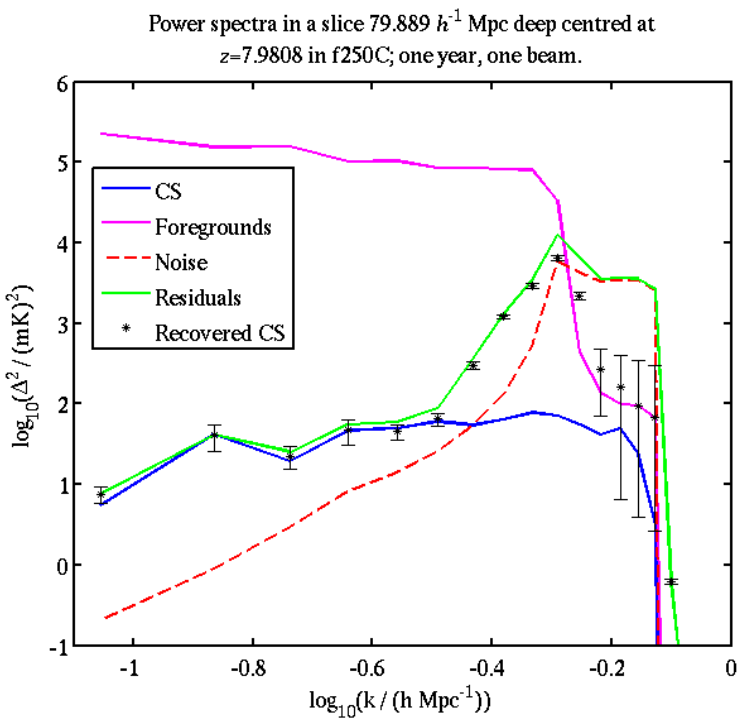
1D power spectrum – high *z*, 1 yr, 1 beam



2D power spectrum – low z, 4 yrs, 4 beams



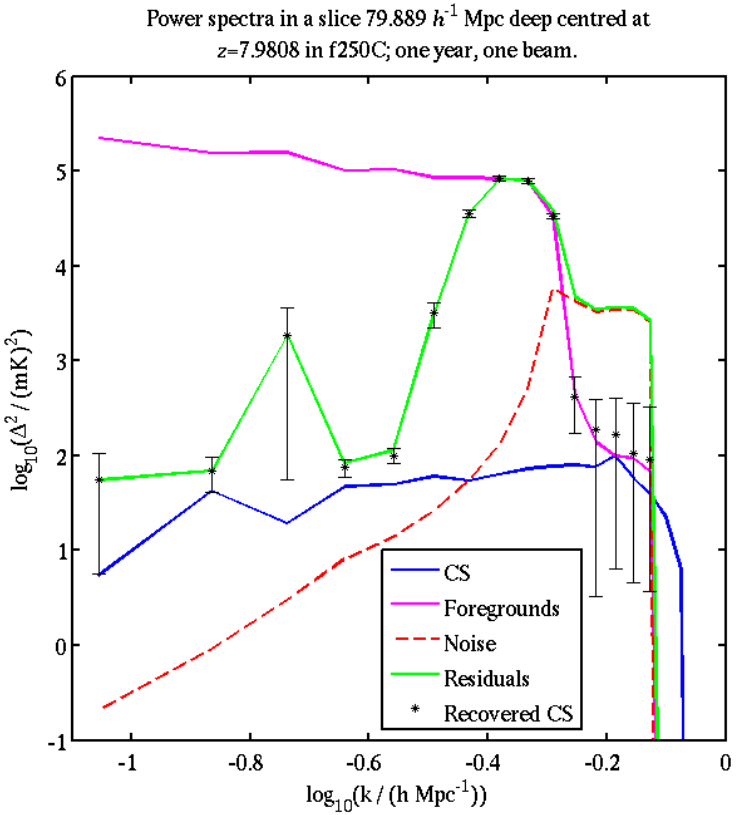
Redshift-dependent *uv* coverage



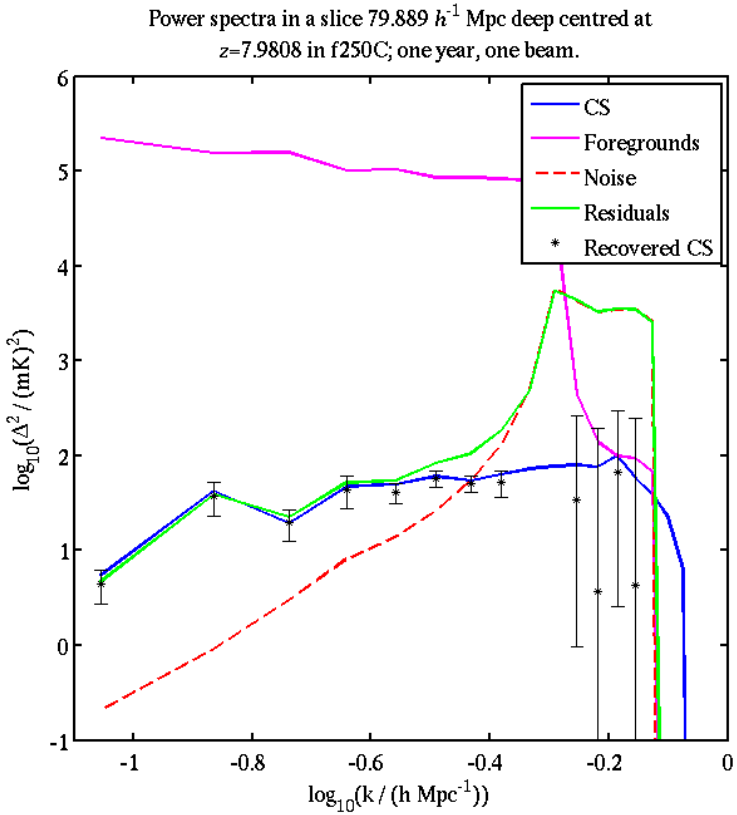
- The *uv* coverage changes as a function of redshift: high ν increases the maximum *k*, moves holes across the *uv* plane and contracts 'frizz' from the PSF across unresolved point sources in the image plane.
- The latter effect introduces spurious small-scale power if we fit the foregrounds in the image plane.
- The most drastic solution is to throw away data until the *uv* coverage is the same at all frequencies.

Fitting in the *uv* plane

Starting from images



Starting from Fourier space



Comments and conclusions

- The power spectrum is a rich source of physical information.
- The different scale-dependence of the foregrounds, noise and cosmological signal may help in extracting the signal.
- Only a rough estimate of the power spectrum is required to extract the evolution of the skewness (after Wiener filtering).
- Foreground fitting is unlikely to be a bottleneck in extraction: we can afford something sophisticated even if it's computationally expensive.
- Realistic levels of noise and diffuse foregrounds do not, in themselves, seem to be a deal-breaker for power spectrum subtraction…
- …but the fitting process introduces biases: can they be corrected for using simulation results?
- As the integration time increases, we can expect continued qualitative improvements in what can be inferred from the data.
- The effects of variable *uv* coverage seem to be manageable with a carefully chosen fitting scheme.

Future work

- How accurately can we really estimate the power spectrum of the noise? Probably need to use the data before they're binned in time and frequency to give a final data cube.
- How is the 'separation of powers' (of μ) affected by the fitting and extraction process?
- Continue to integrate improved models of the instrument, foregrounds, noise and a variety of signal models incorporating larger scales.
- Effect of other error sources: ionosphere, polarization calibration, point source subtraction errors…
- Incorporate signal correlations and a systematic way of choosing the amount of smoothing in the foreground fitting.
- Can we gain from mismatched spatial and frequency resolution?
- Effect of power spectrum errors on recovery of evolution of skewness.
- Full error analysis including cosmic variance; effect of multiple windows.