Halo Formation and Environment in the Millennium Simulation

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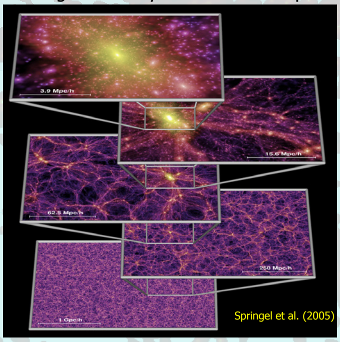
What is the Millennium Simulation?

The recently completed ΛCDM Millennium Simulation (Springel et al. 2005) is the largest cosmological N-body simulation ever performed. Comprising more than 10^{10} particles in a $500h^{-1}$Mpc box, it follows the formation of halos with masses from 5×10^{10} to 10^{15} solar masses in a statistically representative volume. This plot zooms in on a halo which would host a galaxy cluster. The enlargements are by a factor of four each time. While the zoomed out panel shows that the simulation traces the 'cosmic web' over extremely large scales, we see from the final panel that hundreds of independent, gravitationally bound substructures are resolved in a single halo. The colour-coding is by projected density in a slab of thickness $15h^{-1}$Mpc. The cosmological parameters in the simulation, chosen to match the 'concordance cosmology', are as follows:

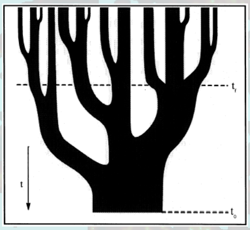


Springel et al. (2005)

- Total matter density at z=0 in units of the critical density, $\Omega_m=\Omega_b+\Omega_{CDM}=0.25$
- Density in baryons, $\Omega_b=0.045$
- Dark energy density, $\Omega_\Lambda=0.75$ (flat ΛCDM)
- Power spectrum index, n=1 and normalisation, $\sigma_8=0.9$

The Formation Redshift of Dark Matter Halos

In this work we define the formation redshift of a dark matter halo as the first redshift slice in the simulation in which the largest progenitor of the halo has half the mass of the final halo. The figure is a schematic representation of the merger tree of a halo, showing how its mass (represented by the thickness of the branch) is built up by the merging of many smaller halos. Time increases towards the bottom of the diagram, and the dotted line shows the time at which we consider the halo to have been formed. This is the definition of formation time most commonly used in analytic studies of halo formation (e.g., Lacey & Cole 1993).

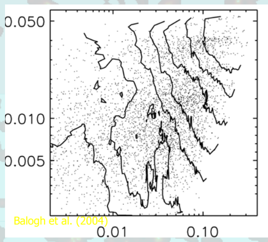


Lacey & Cole (1993)

Why Merger Trees Matter

In hierarchical models, the formation of galaxies is driven by the mergers of the host dark matter halos of the galaxies. Individual galaxies merge and interact, changing their properties – luminosity, morphology, colour and star formation rate for example. Mergers also lead to the formation of groups and clusters of galaxies, within which galaxies may orbit, interact or merge. Therefore the merger histories of dark matter halos affect the observed properties of the galaxy population and the clustering statistics of galaxies.

Why Environment Matters



Balogh et al. (2004)

Galaxy properties are observed to depend on their large scale environment, where environment is usually measured by the number density of some population of galaxies within some nearby region (e.g. Balogh et al. 2004 – this plot from that paper shows contours in the blue fraction of galaxies in the 2dFGRS as a function of local density on two different scales). It is a prediction of hierarchical clustering models that dark matter halos are different in different environments. For example, in denser environments the mass function is biased towards more massive halos. However, it is usually assumed in, for example, semianalytic modelling that the merger history of a dark matter halo depends only on its mass and not on its environment. Previous studies of the environmental dependence of halo formation times (Lemson & Kauffmann 1999) have found no dependence of mean formation time on local density, but at face value this seems inconsistent with the following facts:

- In dense regions, halos tend to be more massive;
- More massive halos tend to have formed more recently.

If halo formation time at a given mass depends on environment this has interesting consequences for our usual models of galaxy formation.

How the Millennium Simulation Helps

- Volume: the box is sufficiently large that there is a reasonable sample of very massive, or very rare and unusual objects, and there are a total of more than 18 million halos above the detection limit of 20 particles in the final output of the simulation. This improves our statistics, especially when studying halos within small mass ranges.
- Dynamic range: a particle mass of 8.6×10^8 solar masses means that all halos expected to host galaxies brighter than $0.1L_*$ are resolved with at least 100 particles.
- Time resolution: with 64 output times (requiring ~20TB of storage) formation times can be determined on a fine grid.

The Marked Correlation Function

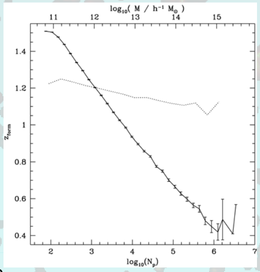
A sensitive new test of environment was proposed recently by Sheth & Tormen (2004). The marked correlation function is a generalisation of the familiar two-point correlation function. For each object, i, we define a 'mark' m_i. The objects considered here are dark matter halos. The mark is taken to be some property of a halo for which we want some measure of the environmental dependence (formation time, for example). Then the marked correlation function, $M(r)$, is defined by

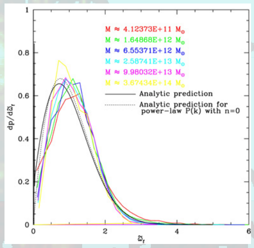
$$M(r)=\frac{\sum_{\{i,j\}} m_i m_j}{\sum_{\{i,j\}} \bar{m}^2}$$

where the sum is taken over all pairs of objects $\{i,j\}$ with separation $r=r_{ij}$, but where the mean is taken over all objects in the sample. Therefore if $M(r)>1$ pairs of objects with separation r tend have a greater value of the mark than average. Note that we do not have to choose some scale on which to define the environment of a halo – instead, the marked correlation function tells us the scale. Note also that because halos of different masses cluster differently, choosing a sample with objects of different masses makes the statistic hard to interpret, though it may tell us about the mass dependence of the mark.

The Scaled Formation Redshift

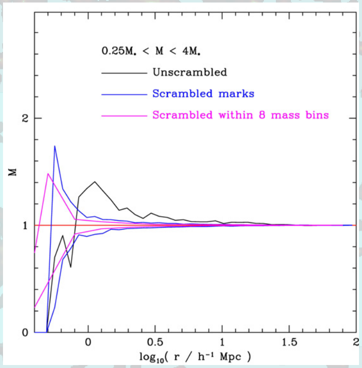
It is helpful to have some measure of formation redshift which we expect to have a distribution independent of the mass of the halo under consideration. For a scale-free initial power spectrum of fluctuations, this measure is provided by the variable $\omega(M,z_{form})$ of Lacey & Cole (1993). They show that extended Press-Schechter theory (Bond et al. 1991) predicts that the distribution of ω is independent of mass. The plot on the right tests this for halos in the Millennium Simulation. The solid line with error bars shows the mean formation redshift of halos as a function of the number of particles in the halo (bottom x-axis) or the mass of the halo (top x-axis). The dotted line shows the mean value of ω for the same halos and demonstrates that most of the mass dependence has been scaled out. One might also worry that the precise distribution of scaled formation times (as well as the mean of this distribution) may affect the results. The coloured lines in the plot on the left show this distribution for halos in various mass bins, while the black lines show the analytic prediction for this distribution. The solid black line shows the prediction using the input power spectrum of the Millennium Simulation. This is plotted for halos of one mass, but theory predicts the distribution is very nearly mass-independent in any case. The dotted black line shows the solution for a power-law initial power spectrum with n=0, and demonstrates that the prediction depends little on the precise initial power spectrum used. In practice, we find our results change very little if we force the distribution of scaled formation redshifts to follow this distribution, independent of mass, or if we instead scale the formation redshift by the mean formation redshift of halos of that mass determined empirically from the simulation.





Results

An example of a marked correlation function of halos from the Millennium Simulation, with scaled formation redshift as the mark, is given in this plot. The black line is the marked correlation function itself, using the subset of halos in the mass range shown. For reference, $M_*=5.9\times10^{13}$ solar masses in this simulation. The blue lines result from assigning each halo the mark of another random halo in the sample and finding $M(r)$, then repeating 100 times and finding the dispersion (as in Sheth & Tormen 2004). The magenta lines show what happens if this shuffling is only done between halos of similar mass. That the black line lies well above these is significant evidence of a signal. It will be interesting to explore what range of masses provides the strongest signal and whether the Millennium Simulation will allow us to similarly find a significant signal in more traditional measures of the dependence of formation time on environment, e.g. as carried out by Lemson and Kauffmann (1999).



Acknowledgements

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