Chaos in the Solar System

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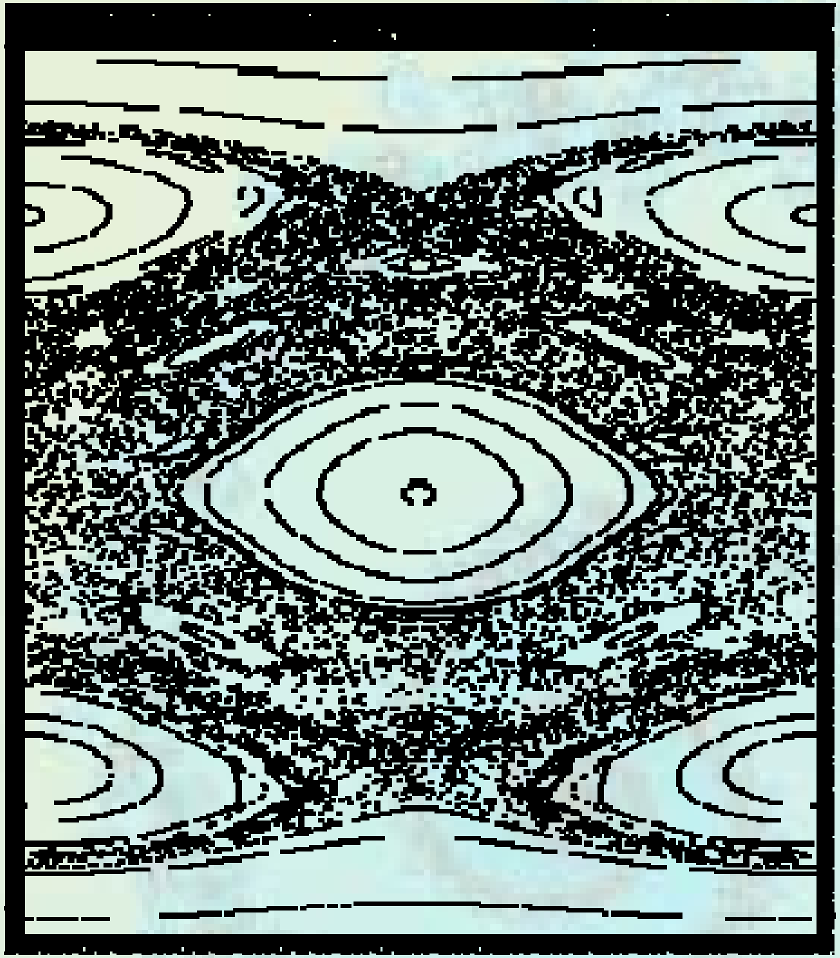
What is chaos?

- Irregular motions arising in deterministic systems…
- …or, an exponential divergence of initially nearby trajectories in phase space.
- Timescale of divergence is the Lyapunov time.
- Examples: driven pendulum; double pendulum; gravitational 3-body problem.

A driven pendulum and its surface of section



Historical perspective

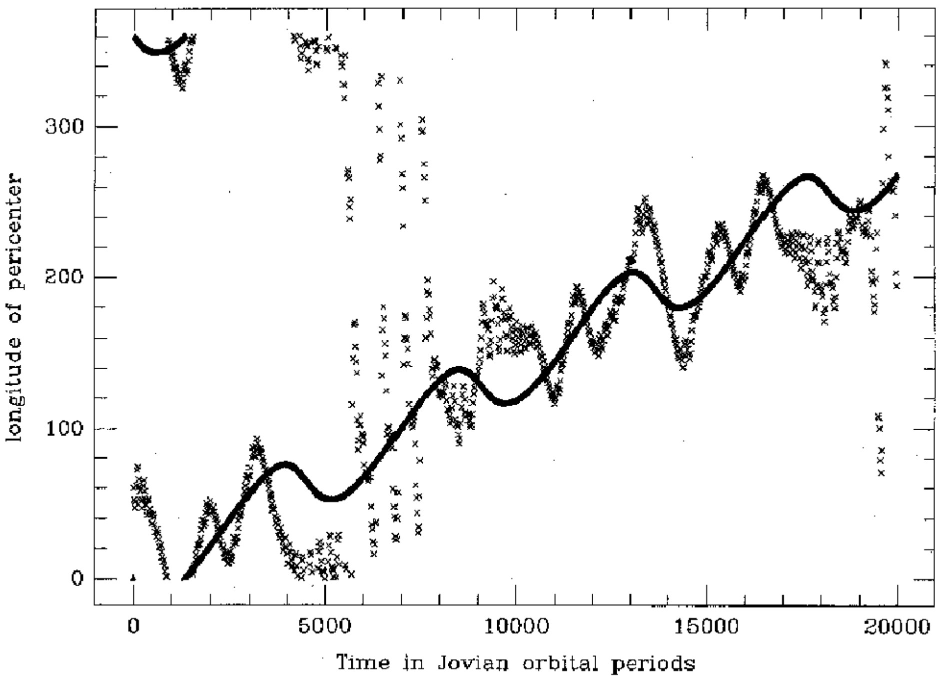
- “Is the Solar System stable?” is an old question.
- Stimulated the early development of topology.
- Detailed study of chaos only possible with the advent of computers.
- Now we can ask, “Is the Solar System chaotic?”

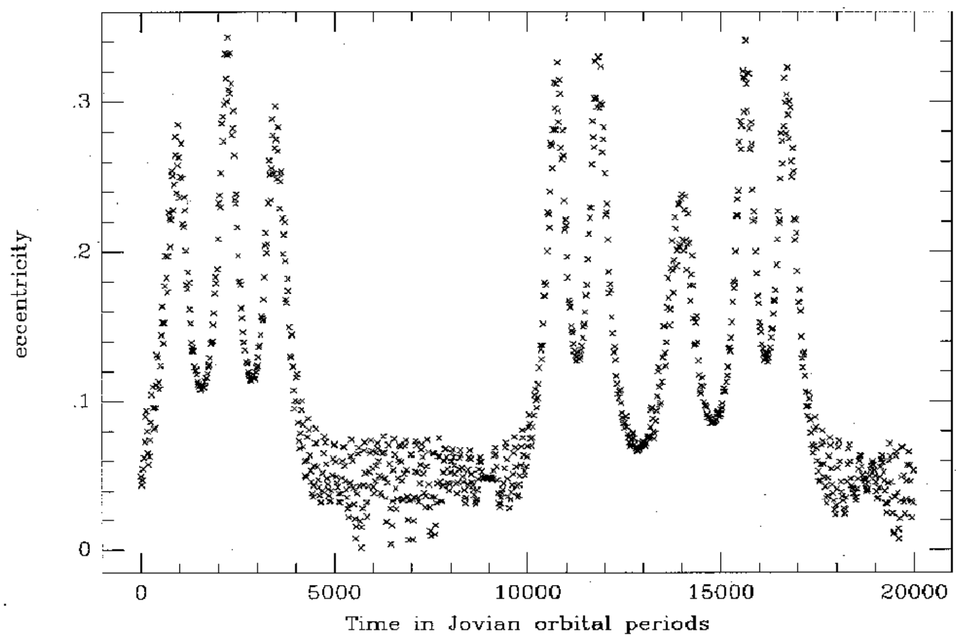
Conditions for chaos

- Stability if there is an integral of the motion for each degree of freedom
- No such integral exists for the three-body problem.
- Non-analytic invariant curves exist for small perturbations with couplings far from resonance (KAM theorem) – not satisfied in the Solar System!
- Chaos arises where neighbouring resonances overlap – this underpins analytic work arising from perturbation theory.

Types of resonance

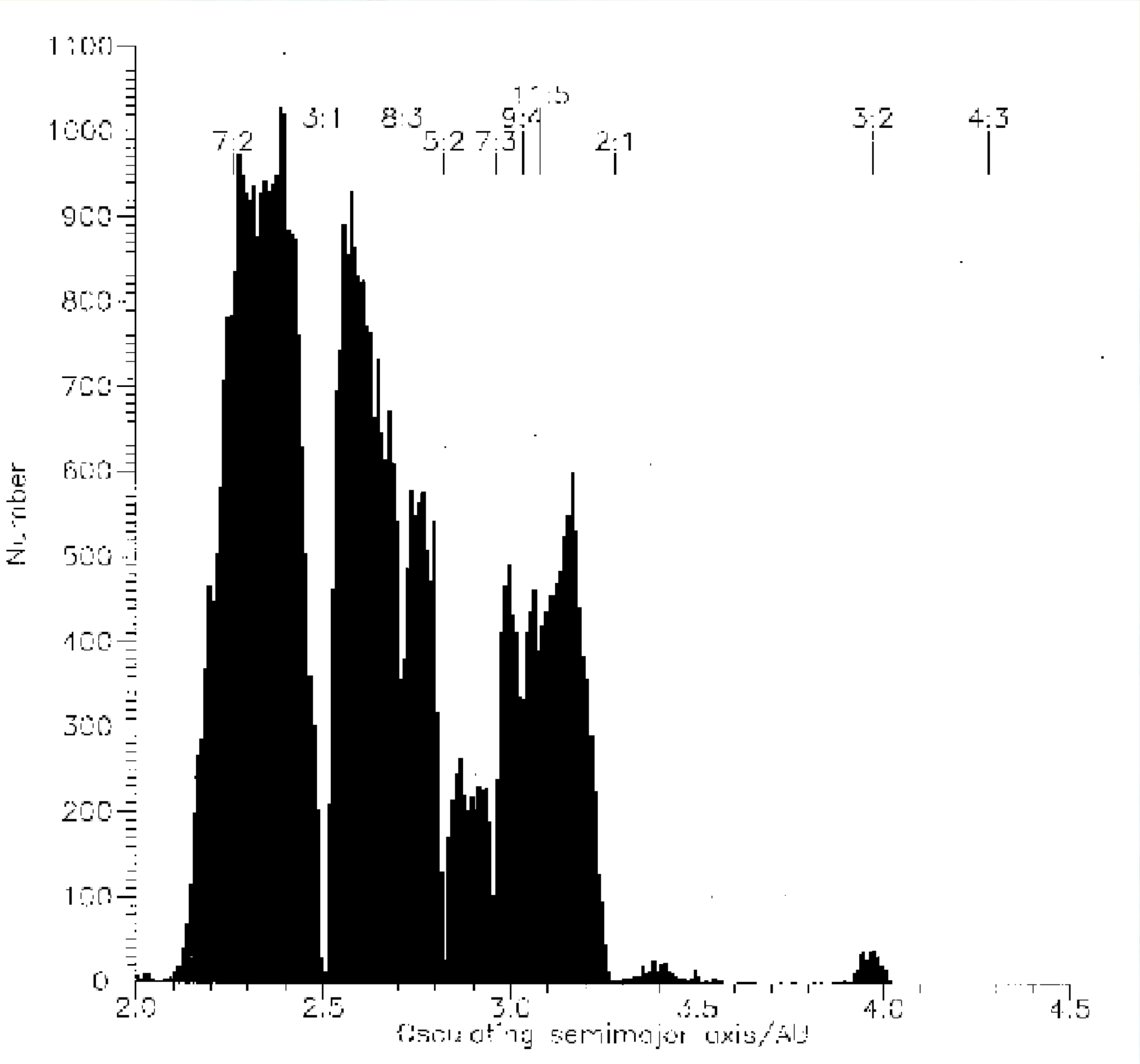
- Mean-motion resonance
  - Orbital periods in integral ratios.
  - For Jupiter, 4:3 and 5:4 mean-motion resonances overlap, as do all higher resonances.
- Secular resonance
  - Equality of apsidal or nodal rates.
  - Important within mean-motion resonances, e.g. Kirkwood gaps in asteroid belt.


longitude of pericenter
300
200
100
0
0
5000
10000
15000
20000
Time in Jovian orbital periods


eccentricity
.3
.2
.1
0
0
5000
10000
15000
20000
Time in Jovian orbital periods

Kirkwood gaps

- Low eccentricity objects increase eccentricity resulting in planet crossing.
- Even objects in stable high eccentricity orbits may be extracted by Mars.
- For the 3:1 resonance timescales are of order 10-100 Myr.
- Resonances more dense past 2:1 – “more chaos”.

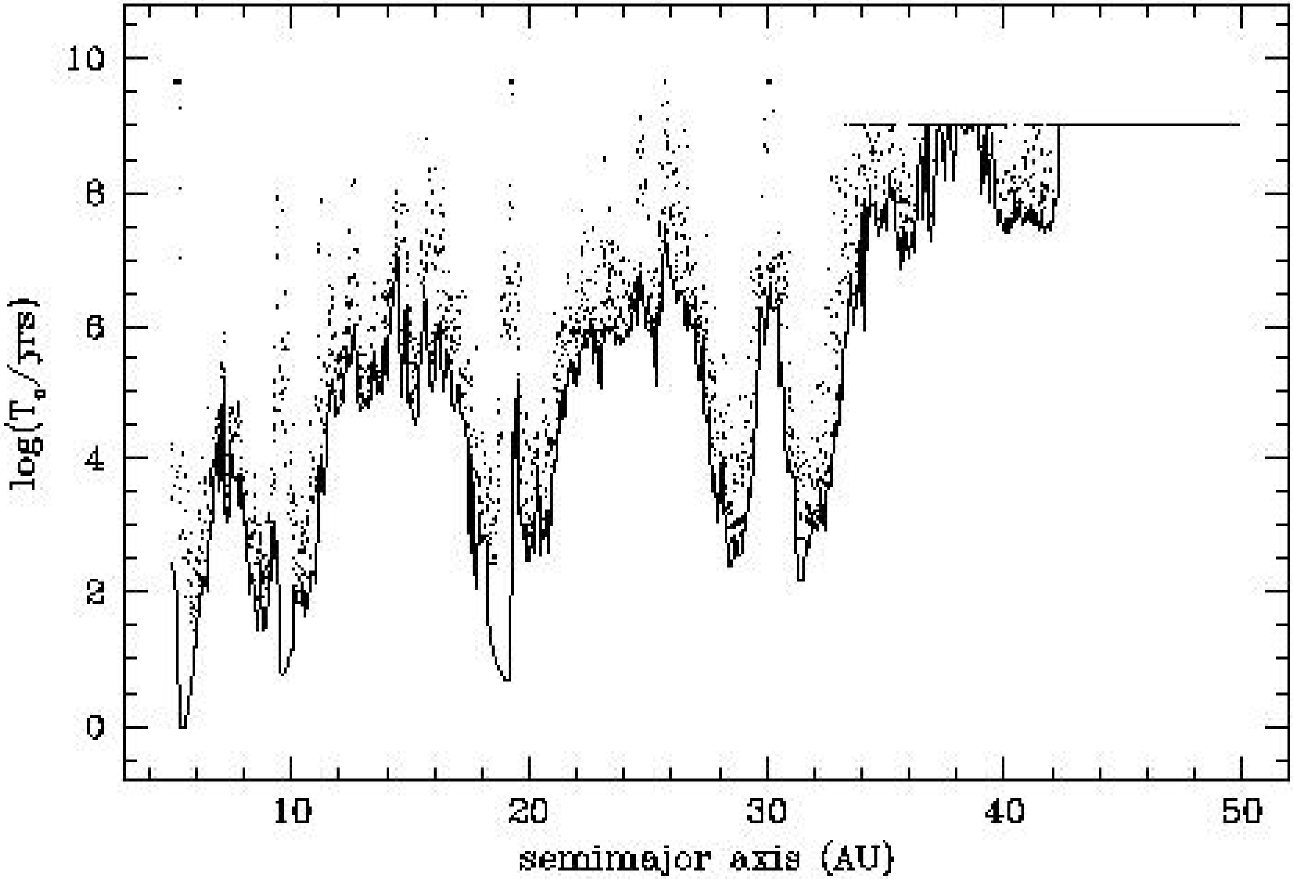

7:2
3:1
8:3
5:2
7:3
9:4
11:5
2:1
3:2
4:3
Number
0
100
200
300
400
500
600
700
800
900
1000
1100
2.0
2.5
3.0
3.5
4.0
4.5
Osculating semimajor axis/AU

Mysterious resonances

- Secondary resonances may contribute to odd behaviour at 2:1 and 3:2
  - Integral ratios of P_{apse} and P_{lib}
  - Needs more realistic models.
- Gaps at fifth and sixth order resonances seem to be too pronounced
  - Ejection timescales of order the age of the Solar System.
  - Other factors may come into play

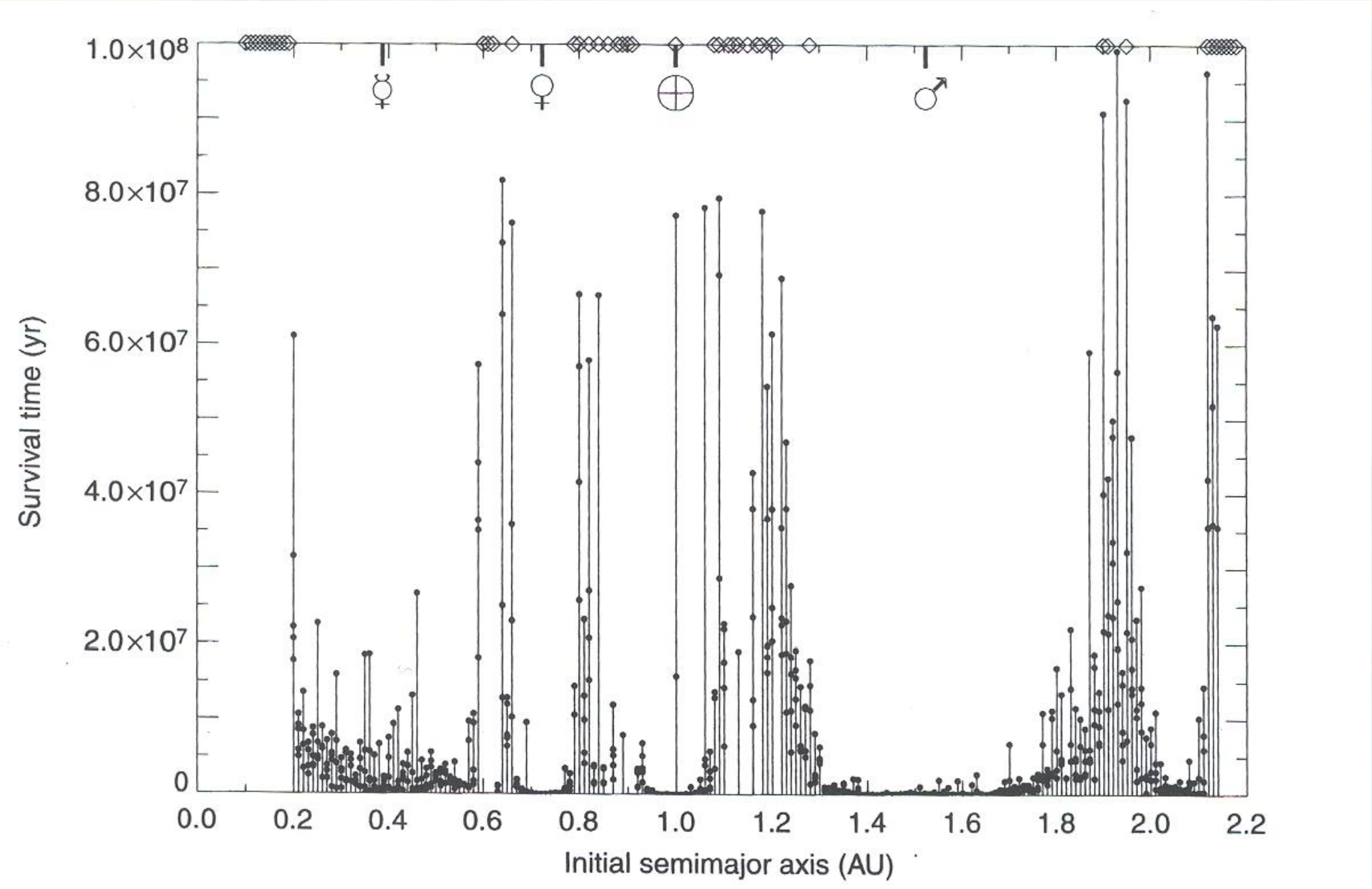
Beyond the asteroid belt

- Low surface density of asteroids between Jupiter and Saturn
  - Most objects ejected very quickly.
  - Very few survive past ~10^7 years.
- Similarly between the other outer planets, but lifetimes increase to ~10^9 years between Uranus and Neptune

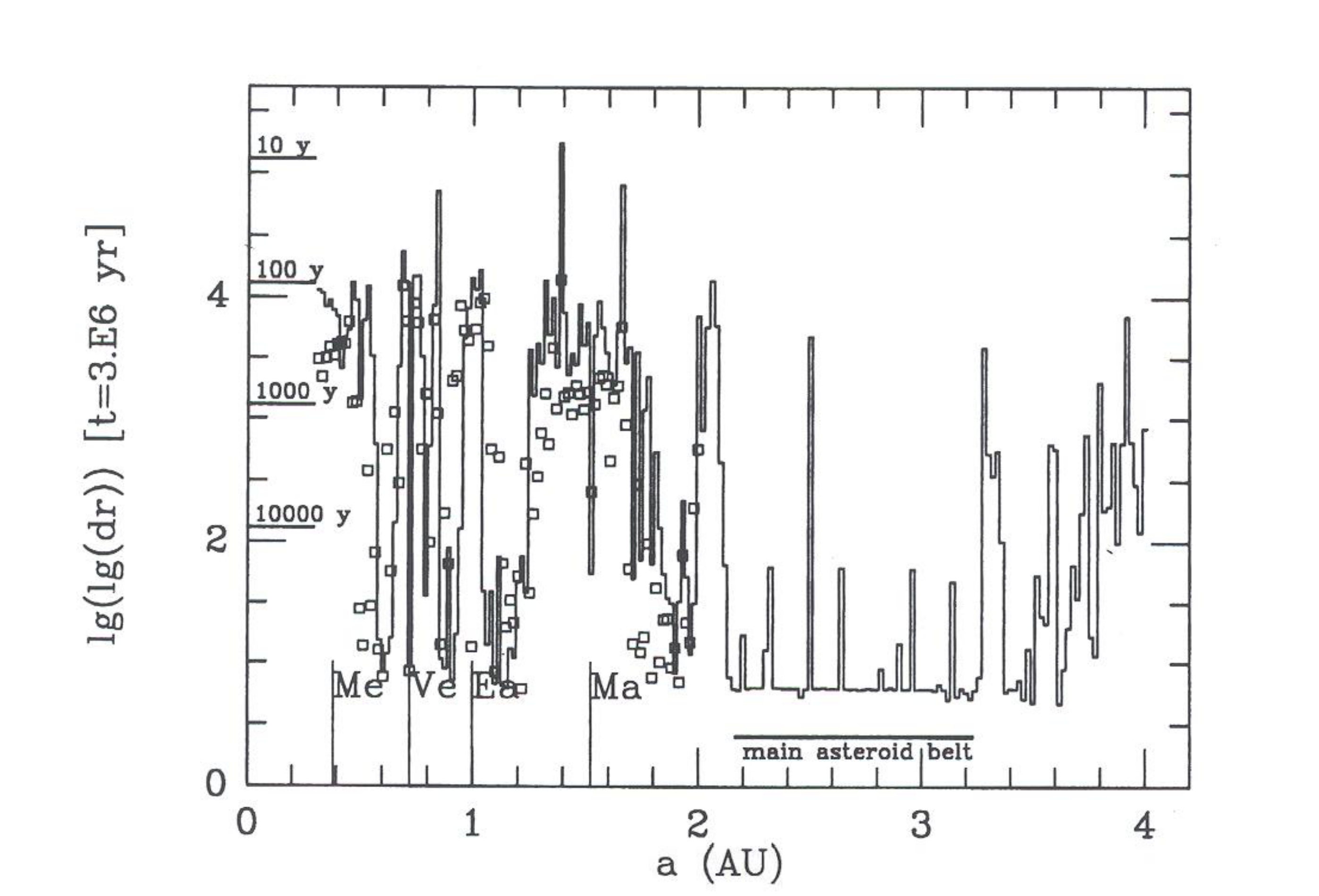


log(T$_o$/yrs)
10
8
6
4
2
0
10
20
30
40
50
semimajor axis (AU)

Inner Solar System

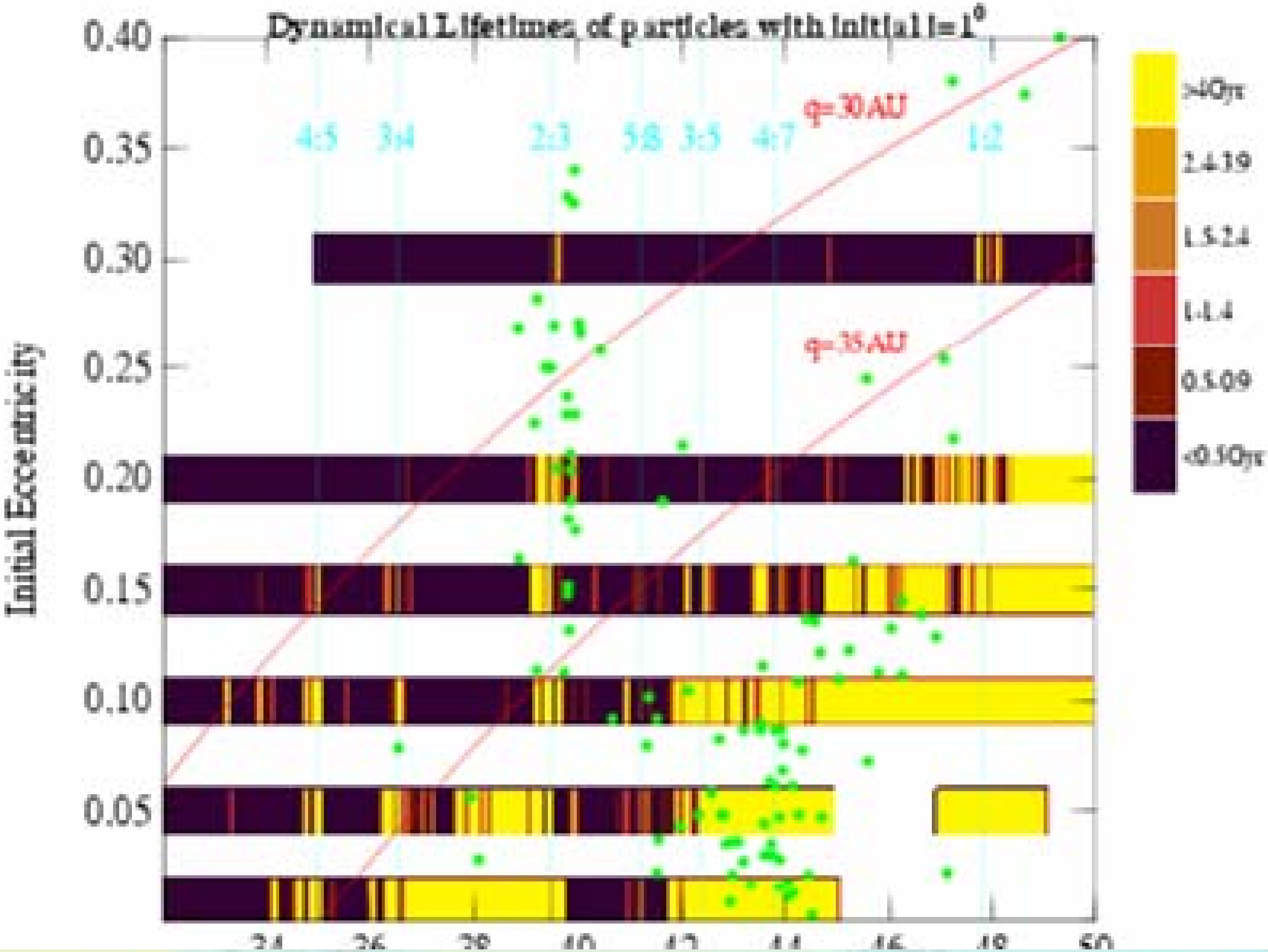


Divergence in phase space



Kuiper Belt

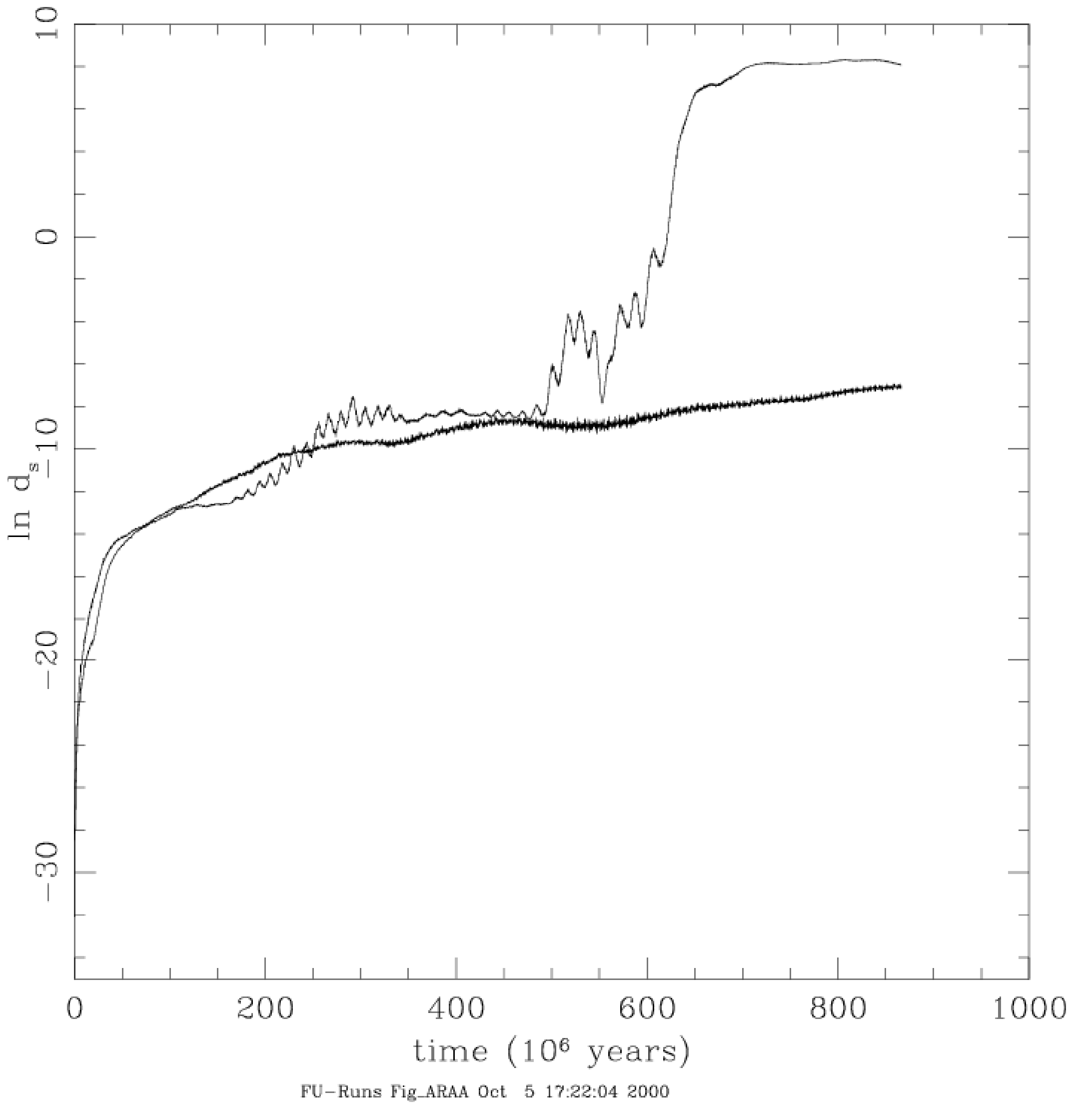
- Disc of material beyond Neptune.
- Stable regions, with ejection times ~10^9 years
- Unstable regions, with ejection times ~10^7-10^9 years.
- Mixture of regions and diffusion between them gives a reservoir of comets.



Dynamical Lifetimes of particles with initial i=1°
Initial Eccentricity
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
4:5
3:4
2:3
5:8
3:5
4:7
1:2
q=30AU
q=35AU
40
42
44
46
48
50
>4Gyr
2.4-3.9
1.5-2.4
1-1.4
0.5-0.9
<0.5Gyr

Are planets safe?

- Big variations in eccentricity of Mercury – diffuses to e=1 in 2000 Gyr
- No dynamical barrier to ejection.
- Only Neptune and Pluto in two-body mean-motion resonance.
- Three-body resonances give ejection times ~10^{18} yrs.
- Variations in obliquity of Mars.


10
0
−10
−20
−30
ln d_s
0
200
400
600
800
1000
time (10^6 years)
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Obliquity of Mars over 400 million years

Conclusions

- The Solar System is chaotic.
- (Crossing time)~(Lyapunov time)$^{1.75}$ but with large spread.
- Two resonances overlap and cause an increase in eccentricity, and eventually orbit crossing
- Would expect stronger chaos in the early Solar System.
- Maverick objects have already been ejected – the Solar System is old, not stable!